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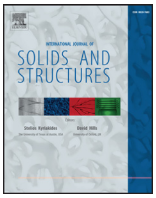

The origin of in-plane stresses in axially moving orthotropic continua[☆]

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ABSTRACT

In this paper, we address the problem of the origin of in-plane stresses in continuous, two-dimensional high-speed webs. In the case of thin, slender webs, a typical modelling approach is the application of a stationary in-plane model, without considering the effects of the in-plane velocity field. However, for high-speed webs this approach is insufficient, because it neglects the coupling between the total material velocity and the deformation experienced by the material. By using a mixed Lagrange–Euler approach in model derivation, the solid continuum problem can be transformed into a solid continuum flow problem. Mass conservation in the flow problem and the behaviour of free edges in the two-dimensional case are both seen to influence the velocity field. We concentrate on solutions of a steady-state type, and study briefly the coupled nature of material viscoelasticity and transport velocity in one dimension. Analytical solutions of the one-dimensional equation are presented with both elastic and viscoelastic material models. The two-dimensional elastic problem is solved numerically using a nonlinear finite element procedure. An important new fundamental feature of the model is the coupling of the driving velocity field to the deformation of the material, while accounting for small deformations of the free edges. The results indicate that inertial effects produce an additional contribution to elastic contraction in unsupported, free webs.

1. Introduction

In the handling of continuous, high-speed webs the origin of in-plane stresses creates a scientific problem, which is not yet completely understood. Especially, the type of the web material has a significant effect on both qualitative and quantitative characteristics of the in-plane stresses. In moving continuous web systems, the tension of the web can usually be controlled in the direction of the transport velocity, the tension being generated by a velocity difference between the starting and ending lines of a free span. At high transport velocities, both web stress and web stability are of concern, not only in this longitudinal direction, but also in the direction perpendicular to the main transport velocity in the plane of the web.

Axially moving materials have many applications in industry, e.g. in paper production, and their mechanics have been studied widely. In the processing of different kinds of thin, laterally moving solid webs, challenges are met, such as the efficiency of production and effects caused by the high processing speed. The first studies of the vibrations of travelling elastic materials date back to the end of the 19th century (Skutch, 1897) and to the middle of the 20th century (Sack, 1954; Archibald and Emslie, 1958). A string model for the moving material was used in all of these studies. Later on, in the 1960s and 1970s, many researchers continued studies on moving strings and beams, concentrating mainly on various aspects of free and forced transverse vibrations (e.g. Mote, 1968, 1972, 1975; Swope and Ames, 1963; Miranker, 1960 and Simpson, 1973).

The stability of small transverse vibrations of travelling two-dimensional rectangular membranes and plates have been studied by Ulsoy and Mote (1982) and Lin (1997). When the web is advancing through a process without external support, the inertial forces depending on the web speed are coupled with web tension. Also the transverse behaviour of the web and the response of the fluid (air) surrounding the web are coupled (see e.g. Chang and Moretti, 1991; Pramila, 1986). Studies modelling the moving web coupled with the surrounding air have been made by Niemi and Pramila (1986), Pramila (1987) and Koivurova and Pramila (1997). In their studies, it was found that the surrounding air significantly reduces the eigenfrequencies and critical velocities of the web, when compared to the vacuum case. Chang and Moretti (1991) studied membranes using potential flow theory, and Banichuk et al. (2010b, 2011b) used the flat panel model coupled with potential flow. This research was extended by Jeronen (2011), where the eigenfrequency spectra were investigated for this model and for the moving string with damping. In Watanabe et al. (2002), two different methods of analysis were

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developed for the phenomenon of paper flutter. One of these was a flutter simulation using a Navier–Stokes code, and the other method was based on a potential flow analysis of an oscillating thin airfoil.

Lin and Mote studied an axially moving membrane in a 2D formulation, predicting the equilibria of the displacement and the stress distribution under transverse loading (Lin and Mote, 1995). Later, they continued studying the wrinkling of axially moving rectangular webs with a small flexural stiffness (Lin and Mote, 1996). They predicted the critical value of the non-linear component of the edge loading after which the web wrinkles, and the corresponding wrinkled shape of the web. It is also known that the lack of web tension will result in a loss of stability of the moving web, which from the application viewpoint, disturbs the required smooth advancing of the web (see e.g. Banichuk et al., 2011a, 2010a). On the other hand, high tension may cause web breaks, which deteriorates production efficiency (see e.g. Robertson, 1963; Banichuk et al., 2013; Sanborn, 1962; Skowronski and Robertson, 1985).

Paper has often been modelled as an orthotropic elastic solid. Elastic constants have been measured for some paper-like materials by Mann et al. (1980) and Baum et al. (1981). Recently, for anisotropic solids, Erkkilä et al. (2013) have studied competent parameters based on modelled stress–strain curves for further construction of a material model. Out-of-plane Poisson ratios, specifically, have been recently studied by Stenberg and Fellers (2002), who reported that paper is an auxetic material: stretching in the machine direction will cause the paper web to thicken in the out-of-plane direction. The relevant Poisson ratio, ν_{13}, is negative, and $|\nu_{13}|$ may be as large as 3.0. Incompressible and slightly compressible orthotropic and transversely isotropic materials have been investigated by Itskov and Aksel (2002), who discovered nontrivial conditions that the elastic constants must satisfy in order to obtain incompressible or slightly compressible behaviour.

Considering wet paper material, the viscoelastic properties play an important role in the behaviour of the web, and thus, need to be included in the model. The first study on transverse vibration of travelling viscoelastic material was carried out by Fung et al. (1997) using a string model. Extending their work, they studied the viscous damping effect in their later research (Fung et al., 1998). Viscoelastic strings and beams have recently been studied extensively, see e.g. Marynowski and Kapitaniak (2007) and Zhang and Chen (2005). Oh et al. studied critical speeds, eigenvalues and natural modes of the transverse displacement of axially moving viscoelastic beams using the spectral element method (Lee and Oh, 2005; Oh et al., 2004). Chen and Zhao (2005) presented a modified finite difference method to simplify a non-linear model of an axially moving string. They studied numerically the free transverse vibrations of both elastic and viscoelastic strings. Chen and Yang studied free vibrations of viscoelastic beams travelling between simple supports with torsion strings (Chen and Yang, 2006). They studied the viscoelastic effect by perturbing the similar elastic problem and using the method of multiple scales. Very recently, Yang et al. studied vibrations, bifurcation, and chaos of axially moving viscoelastic plates using finite differences and a non-linear model for transverse displacements (Yang et al., 2012).

Marynowski and Kapitaniak studied differences between the Kelvin–Voigt and Burgers models in the modelling of the internal damping of axially moving viscoelastic beams. They found out that both models gave accurate results with small damping coefficients, but with a large damping coefficient, the Burgers model was more accurate (Marynowski and Kapitaniak, 2002). In 2007, they compared the models with the Zener model studying the dynamic behaviour of an axially moving viscoelastic beam (Marynowski and Kapitaniak, 2007). They found out that the Burgers and Zener models gave similar results for the critical transport speed whereas, the Kelvin–Voigt model gave a greater critical speed compared to the other two models.

The origin and structure of the in-plane stress and strain distribution in a moving solid web seems to be an exceptionally unknown area. The models used with web materials are often based on assumptions of isotropic or orthotropic material properties (see e.g. Baum et al., 1981; Thorpe, 1981). The material is considered as viscoelastic or viscoplastic, but in the models, there is usually no coupling between the in-plane strain and the web velocity effects (see e.g. Hauptmann and Cutshall, 1977; Pecht and Johnson, 1985; Uesaka et al., 1980).

Time-dependent, in-plane vibrations of a moving continuous membrane were studied by Shin et al. (2005). In their work, in-plane vibration modes of an isotropic web were studied between the traction lines. Also Guan et al. have studied viscoelastic web behaviour in both steady state and unsteady state cases (Guan et al., 1995; 1998). In the recent years, the topic of supercritically moving materials has received attention; for example, see Ding et al. (2012), where steady-state periodic responses regarding time-harmonic forced vibrations of supercritically moving viscoelastic beams were studied using multiscale analysis and finite differences. Note, however, that in the present study, we will concentrate on the subcritical regime, which is more relevant for paper production.

Traditionally, the partial time derivative has been used instead of the material derivative in the viscoelastic constitutive relations, but Mockensturm and Guo suggested that the material derivative should be used (Mockensturm and Guo, 2005). They studied non-linear vibrations and the dynamic response of axially moving viscoelastic strings. Kurki and Lehtinen also independently suggested that the material derivative should be used in the constitutive relations, in their study concerning the in-plane displacement field of a travelling viscoelastic plate (Kurki, 2005; Kurki and Lehtinen, 2009).

In a study by Chen et al., the material derivative was used in the viscoelastic constitutive relations (Chen et al., 2008). They studied parametric vibration of axially accelerating viscoelastic strings. Chen and Ding studied the stability of axially accelerating viscoelastic beams using the method of multiple scales, and the material derivative was used in the viscoelastic constitutive relations (Ding and Chen, 2008). Chen and Wang studied the stability of axially accelerating viscoelastic beams using asymptotic perturbation analysis and the material derivative in the viscoelastic relations (Chen and Wang, 2009). The material derivative was also used in a recent paper by Chen and Ding, where the dynamic vibration response of axially moving viscoelastic beams was studied (Chen and Ding, 2010). A non-linear model was used, taking into account the coupling of the transverse displacement with the longitudinal (in-plane) displacement. However, the transverse behaviour of the beam was their main focus.

In this paper, we propose to modify the classical two-dimensional model of a moving viscoelastic web by accounting for the coupling between the velocity field and the in-plane strain. A two-dimensional, thin open loop (non-conservative system) made of an orthotropic membrane is stretched using a relative speed difference between the traction lines. The orthotropic viscoelastic material assumption is applied, using a viscoelastic model of the Kelvin–Voigt type. The axial motion is accounted for by using a moving reference state that translates axially at constant velocity (see e.g. Tannehill et al., 1997; Koivurova and Salonen, 1999). This method handles the behaviour of a solid moving web using a control volume approach similar to the treatment of fluid flows. Preliminary one-dimensional studies have been reported in the paper (Kurki et al., 2012). In the present paper, the steady state of the two-dimensional moving continuum, in the pure elastic case, is solved using the nonlinear finite element method. An important new fundamental feature of the considered model is the coupling of the driving velocity field to the deformation of the material, while accounting for small deformations of the free edges.