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One dimensional constitutive model with transformation surfaces for phase transition in shape memory alloys considering the effect of loading history

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ABSTRACT

Existing constitutive models for Shape Memory Alloys (SMAs) assume that both forward and reverse transformations occur when the thermodynamic driving force reaches a specific amount regardless of loading history. In this article, these assumptions are examined, and some cases are introduced where these models predict contradictory results. The effects of initial martensitic volume fraction on both forward and reverse transformations are shown by carrying out simple tensile tests on SMA wires. In line with these experiments, a one-dimensional constitutive model with new transformation conditions is proposed phenomenologically in order to model pseudoelastic behavior. The constitutive model is proved to be consistent with the theory of continuum mechanics. New transformation surfaces are introduced to govern transformations start conditions, rather than using preexisting common phase diagrams. As a result, history-dependent transformation start temperatures are determined. The obtained experimental stress-strain diagrams, available DSC test results, and experimental strain-temperature responses are used to validate the proposed model. It is shown that loading history affects transformation start conditions.

1. Introduction

Shape Memory Alloys (SMAs) are a branch of smart materials that can return to their initial shape after being subjected to inelastic deformations. The mechanism underlying this property is based on a martensitic transformation between austenite and martensite phases. At zero stress, by heating, transformation from martensite to austenite begins at A_s (austenite start temperature) and ends at A_f (austenite final temperature). When austenite is cooled, transformation to martensite starts at M_s (martensite start temperature) and finishes at M_f (martensite final temperature). The obtained phase is called twinned martensite. If martensite is formed in the presence of stress, detwinned martensite along with macroscopic strains are obtained. This strain will disappear when the material is heated above A_f. This property is named Shape Memory Effect (SME) (Brinson 1993) since the material goes back to its initial configuration. At a constant temperature above A_f, during loading, austenite transforms to martensite but returns to its initial phase and shape in the course of unloading. This phenomenon is named Pseudoelasticity (PE) (Brinson 1993). These two unique properties allow shape memory alloys to have extensive applications such as prosthetic hand or active bending catheter in biomedical engineering (Cismasiu 2010). In non-medical applications, SMAs can be used as couplings and fasteners (Humbeeck 1999), tendons in structures (Song et al. 2006), actuators (Mammano and Dragoni 2011), micro actuators (Reynaerts et al. 1999), or mini actuators (Nespoli et al. 2010). Moreover, they have the potential to be used as kernel components for seismic protection devices (Dolce and Cardone 2001).

Due to the wide applications of SMA wires, several 1-D constitutive laws have been developed to study the thermomechanical behaviors of shape memory alloys. The earliest models are based on phenomenological approaches. Tanaka et al. (1982, 1986) introduced martensitic volume fraction as an internal variable and used an exponential form for kinetic laws to calculate the amount of the martensitic volume fraction. Liang and Rogers (1990) utilized the initial martensitic volume fraction in their kinetics laws to consider the loading history in the pseudoelastic response of an SMA. Brinson (1993, 1996) divided the martensitic volume fraction into temperature- and stress-induced parts to study shape memory effect as well as pseudoelasticity.

In the existing models, it is assumed that transformation temperatures and loading history are independent. However, Buravalla and Khandelwal (2011) carried out experiments and reported different findings. In their experiments, during reverse transformation (M to A), an SMA is arrested at a temperature between A_s and A_f (in this

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paper, this temperature is denoted by $A*$). Afterward, the specimen is cooled to a temperature less than A_s but greater than M_s. In the subsequent heating cycle, it was observed that transformation does not start at A_s; rather, it starts almost at the arrested temperature $A*$. Similarly, for any interruption during forward transformation (A to M), a change in the martensite start temperature is observed. Moreover, if the latter heating cycle continues to a temperature above A_s, the reverse transformation does not start at A_s, but it starts at a greater temperature. Buravalla and Khandelwal (2011) realized similar phenomena in the presence of stress as well. They found empirical strain-temperature responses at a constant stress and showed that, after any interruptions during phase transformation, the subsequent transformation begins at the arrested temperature. According to these findings, it can be concluded that transformation start conditions are influenced by loading history. These results are all different from the above-mentioned assumptions made in the existing constitutive models including those proposed by Brinson (1993, 1996) and Liang and Rogers (1990).

Bekker and Brinson (1997, 1998) introduced switching points on a stress-temperature loading path in phase diagram at which martensitic volume fraction starts to change. It is expressed that phase transformation continues as long as the projection of loading path on the normal direction of the phase diagram is a positive value. By this approach, they added a new condition to the kinetic laws for predicting the beginning of phase transformation under any arbitrary loadings. Based on this model, Buravalla and Khandelwal (2011) considered a set of projection parameters and stated that, after any interruption during phase transformation, a new transformation starts when the new projection at new switching point is smaller than the minimum of the former projection parameters in the set. This new assumption is added to Brinson model as another necessary condition for the beginning of transformations. However, this model cannot predict the shifts in A_s due to the existence of loading history. Jiang et al. (2012) carried out strain-temperature experiments in which interruptions were imposed before transformations were complete. They showed that, after any interruption, new transformation does not start at the beginning of the last one. Banerjee (2012) used both Tanaka equations (1982, 1986) and Bekker and Brinson approach (1997, 1998) to propose an algorithm for the programming of SMA wires as actuators under any arbitrary loading.

In this article, since the goal is to introduce an enhanced constitutive model for phase transformation in shape memory alloys in the simplest form, the Liang and Rogers model (1990) is used. Special heating-cooling cycles are first studied to show that initial martensitic volume fraction affects transformation start conditions. Such effects are also shown to exist in the presence of stress by carrying out simple tensile tests on Nitinol wires. Accordingly, a phenomenological approach is applied to obtain enhanced transformation start conditions as well as a new constitutive model for pseudoelasticity in the simplest form. Liang and Rogers phase diagram is generalized to transformation surfaces in which loading history is considered as one of the main parameters affecting the transformation start conditions. Moreover, history-dependent transformation start temperatures are obtained by using the present approach. This enhanced model is also derived in a continuum framework. Numerical results are compared with the obtained experimental findings for simple tensile test of a Nitinol wire with some interruptions during unloading. A tensile stress-strain response containing several interruptions during loading, a strain-temperature response at a constant stress, and DSC results with some interruptions, done by Buravalla and Khandelwal (2011), are also compared with the predictions of the newly proposed model. A good agreement is shown to exist between the numerical and empirical results indicating the validity of the present model.

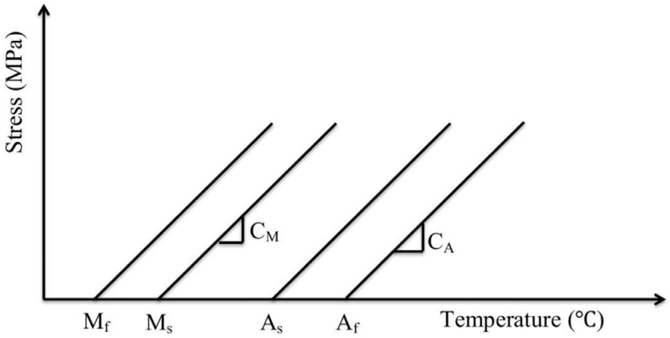

Fig. 1. Stress-temperature phase diagram in Liang and Rogers model.

2. Effect of initial martensitic volume fraction on the transformation start conditions

In order to propose an enhanced constitutive model for phase transformation in SMAs in a simple form and to examine the effects of loading history on transformation start conditions, Liang and Rogers model (1990) is considered. Referring to the phase diagram shown in Fig. 1, Liang and Rogers kinetic law for the martensitic volume fraction ξ is:

$$\begin{cases} \xi = \dfrac{1-\xi_0}{2}\cos\left\{a_M\left(T - M_f - \dfrac{\sigma}{C_M}\right)\right\} + \dfrac{1+\xi_0}{2};\ A\,\text{to}\,M \\ C_M(T-M_s) < \sigma < C_M\left(T-M_f\right) \\ \xi = \dfrac{\xi_0}{2}\left\{\cos\left\{a_A\left(T - A_s - \dfrac{\sigma}{C_A}\right)\right\} + 1\right\};\ M\,\text{to}\,A \\ C_A\left(T-A_f\right) < \sigma < C_A(T-A_s) \end{cases} \tag{1}$$

where ξ_0 is initial martensitic volume fraction prior to the current transformation, C_M and C_A are respectively slopes of the forward and reverse transformation strips, and the parameters a_M and a_A are defined as $a_M = \frac{\pi}{M_s - M_f}$ and $a_A = \frac{\pi}{A_f - A_s}$.

It is assumed in Eq. (1) that transformation start conditions and loading history are independent. Accordingly, if austenite is cooled to the temperature of $\frac{M_s+M_f}{2}$ then heated to a temperature above M_s (but below A_s) followed by cooling again until $\frac{M_s+M_f}{2}$, a new transformation still begins in M_s regardless of the thermal loading history. If this cooling-heating cycle is successively repeated as shown in Fig. 2(a), the martensitic volume fraction at the end of each cycle increases as shown in Fig. 2(b). It is seen that the amount of martensitic volume fraction reaches 1 after around 8 cycles without crossing M_f. This paradox is observed since the amount of martensitic volume fraction at the end of each cycle becomes the initial martensitic volume fraction for the next one Additionally, it is assumed that transformation begins at M_s regardless of the thermal loading history and that the amount of martensitic volume fraction is dependent on ξ_0.

The same procedure can be considered for reverse transformation as well. The schematic diagram for heating-cooling cycles between a temperature greater than M_s (but below A_s) and $\frac{A_s+A_f}{2}$ is shown in Fig. 3(a), and variations of martensitic volume fraction are shown in Fig. 3(b). Again, it is seen that the material reaches the full austenite phase at a temperature less than A_f.

Reasoning behind these behaviors can be stated according to the fact that transformation start conditions and loading history in such models are independent.