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An overview of different asymptotic models for anisotropic three-layer plates with soft adhesive

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ABSTRACT

We give an overview of the possible asymptotic models for a layered plate with soft adhesive. More specifically, we study the mechanical behavior of an anisotropic nonhomogeneous linearly elastic three-layer plate with soft adhesive, including the inertia forces, by means of the asymptotic expansion method. By defining a small parameter ε, associated with the size and the stiffness of the intermediate layer, we derive various limit models and their corresponding limit problems, by varying the thickness and rigidity ratios of the adherents and the adhesive layers.

1. Introduction

The modeling of complex structures obtained by joining simpler elements with highly contrasted geometric and/or material characteristics represents a source of a variety of problems of practical importance in all fields of engineering. The geometrical complexity of a multilayer structure requires an effort to deduce simplified mathematical models: these models must take into account the presence of different sizes and stiffnesses among each constituent of the structure. In the present work, we focus our attention to a particular structural assembly consisting in two plates bonded together by a soft adhesive middle layer.

This paper attempts to give a complete spectrum of the possible reduced models for a generic three-layer plate with soft adhesive, comprising all the possible choices of thickness and rigidity ratios between the intermediate layer and the surrounding plate-like bodies. These models are derived by means of the asymptotic expansion method. The asymptotic methods allow to determine the so-called limit model without any a priori assumptions on the displacements and/or stress field of the resulting limit models, by considering only the geometrical and mechanical peculiarities of the structure, such as the small thickness or the elastic moduli ratios of the different layers constituting the multilayer assembly.

More specifically, we analyze the time-dependent mechanical behavior of an anisotropic non homogeneous linearly elastic three-layer plates with soft adhesive. By defining a small parameter ε, which will tend to zero, we suppose that the thickness of the upper and lower plate-like bodies depends linearly on ε, while the thickness of the middle layer has order of magnitude ε^n, $n \in \mathbb{N}$, $n \geq 1$. Moreover, we assume that the elastic coefficients of the top and bottom plates are independent of ε , while the elastic moduli of the adhesive varies with ε^p, $p \in \mathbb{N}$, $p \geq 1$. Then, we derive a series of limit models by taking into account all the possible choices of the magnitudes $\{n, p\}$.

The asymptotic analysis has been successfully employed not only to formally justify classical theories of beams, plates and shells (see, e.g., Ciarlet, 1997), in the framework of linear and nonlinear elasticity, but also to deduce rational simplified models of structural elements bonded together with a thin elastic interphase, which represents the most peculiar bonded joint between two media. The actual computation of the solution of this problem is quite difficult, even if numerical methods are employed: this is mostly due to the thinness of the adhesive, which requires a fine mesh and, hence, an increase of the degrees of freedom of the system. Moreover, the adhesive has usually a different rigidity with respect to the adherents and this causes numerical instabilities in the stiffness matrix. The previous difficulties can be overcome by introducing a reduced model of the adhesive which can be treated as an interface, by assuming, for instance, that the upper and lower bodies are linked by a continuous distributions of springs. This model has been initially proposed in the milestone paper by Goland and Reissner (1944).

Within the theory of elasticity, the asymptotic analysis of a thin elastic interphase between two elastic materials has been deeply investigated through the years, by varying the rigidity ratios between the thin inclusion and the surrounding materials and by considering different geometry features. It is worth mentioning the pioneering work by Acerbi et al. (1988) on the variational behavior of the elastic energy of a thin inclusion using Γ-convergence. Moreover, we

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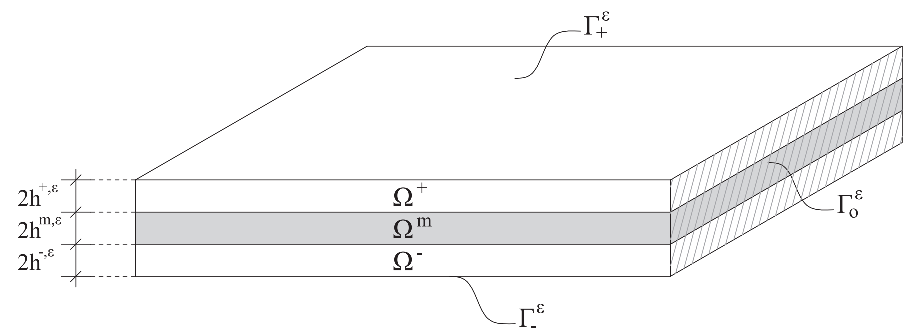

Fig. 1. The reference configuration of the layered plate.

refer to the contributions by Licht and Michaille (1997), Abdelmoula et al. (1998), Geymonat et al. (1999), Klarbring (1991), Klarbring and Movchan (1998) and Krasucki et al. (2004), for mathematical models for linear and nonlinear bonded joints with a soft thin adhesive and, also, to the papers Bessoud et al. (2009, 2008, 2011), Lebon and Rizzoni (2010), 2011) and Lebon and Zaittouni (2010), for the case of multimaterials with thin plate-like and shell-like inclusions with high rigidity. In those papers, existence and uniqueness of the solution of the limit problem and weak, strong and Γ-convergence results have been fully described.

The mechanics behind the junction of two plates has been studied in several works in a rigorous mathematical framework: for instance, Geymonat and Krasucki (1997) and Zaittouni et al. (2002) analyzed two Kirchhoff–Love isotropic plates joint together by a thinner isotropic adhesive, by varying the order of magnitude of the elastic moduli of the intermediate layer; more recently, Serpilli (2005) and Serpilli and Lenci (2008) analyze the mechanical behavior of three different two-dimensional isotropic layered strips through the asymptotic methods: namely, the case of comparable thicknesses and weak adhesive (analogous to the case $n = 1$ and $p = 2$, presented and commented in Section 3.2), the case of comparable thicknesses and comparable rigidities, and, finally, the case of a thinner and stiffer adhesive (these two cases are not treated in the present paper). Besides, in Serpilli and Lenci (2012), the authors study the linear dynamics of a two-dimensional three-layer strip, by characterizing the limit natural high, low and mean frequencies. In these papers the authors recover one-dimensional simplified models, starting from two-dimensional layered strips. While, in the present work, starting from a three-dimensional stack of plates, we derive two-dimensional limit models. Another important contribution is the paper by Åslund (2005), in which the author performs an asymptotic analysis starting from a three-dimensional geometrical configuration: by defining a small parameter ε, the author considers a three-layer plate-like body constituted by two top and bottom plates of thickness ε, bonded by superposition with an adhesive layer of thickness ε^2. The three layers are made of Saint-Venant–Kirchhoff materials and the Lamé's constants of the adhesive have order of magnitude ε^3 with respect to those of the upper and lower bodies. A distinguishing feature of the resulting limit model is that the shear forces dominate in the adhesive, whose membrane displacements depend on the gap of the membrane displacements at the interface between the upper and lower plates. The paper by Schmidt (2008) is a remarkable work that deserves to be mentioned: indeed, the author analyzes the mechanical behavior of two bonded plates with a thin soft isotropic adhesive via the asymptotic expansion methods and derives a two-dimensional surface model for this particular joint. Different cases of rigidity ratios between the adherents and the adhesive have been studied and, moreover, higher-order corrector terms of the asymptotic expansion have been characterize in order to improve and make an error estimate of the solution of the derived models. Finally, it is also noteworthy the paper by Licht (2007), in which the author considers two linearly elastic plates linked by a soft linearly elastic isotropic adhesive: the assembly is made by abutting or by superposition. The reduced models are derived by means of a two small parameters asymptotic analysis, with formal convergence results, and they correspond to bonding two Kirchhoff–Love plates by a mechanical constraint depending on the magnitude of the chosen parameters.

The layout of the paper is as follows. In Section 2, we define the statement of the problem and we perform the asymptotic analysis by defining the dependences on ε of the geometrical and mechanical quantities. In Section 3, we derive the asymptotic models by fixing $n = 1$ and by varying the magnitude $p \geq 1$. In Section 4, we deduce the asymptotic models by fixing $n \geq 2$ and by changing the magnitude $p \geq 1$. In Section 5, we discuss the obtained results in an extensive way and, finally, in Section 6, we give some concluding remarks to the paper.

2. Statement of the problem

In the sequel, Greek indices range in the set {1, 2}, Latin indices range in the set {1, 2, 3}, except *m, n, p*, and the Einstein's summation convention with respect to the repeated indices is adopted. Let $\omega \in \mathbb{R}^2$ be a smooth domain in the plane spanned by vectors $\mathbf{e}_\alpha$, let γ_0 be a measurable subset of the boundary γ of the set ω, such that length $\gamma_0 > 0$, and let $0 < \varepsilon < 1$ be a dimensionless *small* real parameter which will tend to zero. For each ε, we define

$$\begin{aligned}
&\Omega^{m,\varepsilon} := \omega \times \mathcal{I}^{m,\varepsilon},\ \ \Omega^{+,\varepsilon} := \omega \times \mathcal{I}^{+,\varepsilon},\ \ \Omega^{-,\varepsilon} := \omega \times \mathcal{I}^{-,\varepsilon},\\
&\mathcal{I}^{m,\varepsilon} := (-h^{m,\varepsilon}, h^{m,\varepsilon}),\ \ \mathcal{I}^{+,\varepsilon} := (h^{m,\varepsilon}, h^{m,\varepsilon} + 2h^{+,\varepsilon}),\\
&\mathcal{I}^{-,\varepsilon} := (-h^{m,\varepsilon} - 2h^{-,\varepsilon}, -h^{m,\varepsilon}),\\
&\mathcal{I}^{\varepsilon} := (-h^{m,\varepsilon} - 2h^{-,\varepsilon}, h^{m,\varepsilon} + 2h^{+,\varepsilon})\\
&\Gamma_0^{\varepsilon} := \gamma_0 \times \mathcal{I}^{\varepsilon},\ \ \Gamma_{\pm}^{\varepsilon} := \omega \times \{\pm(h^{m,\varepsilon} + 2h^{\pm,\varepsilon})\},\\
&S_{\pm}^{\varepsilon} := \omega \times \{\pm h^{m,\varepsilon}\}.
\end{aligned}$$

Hence the boundary of the set $\Omega^\varepsilon := \Omega^{+,\varepsilon} \cup \Omega^{m,\varepsilon} \cup \Omega^{-,\varepsilon}$ is partitioned into the lateral surface $\gamma \times \mathcal{I}^\varepsilon$ and the upper and lower faces Γ_+^ε and Γ_-^ε, and the lateral surface is itself partitioned as $\gamma \times \mathcal{I}^\varepsilon = (\gamma_0 \times \mathcal{I}^\varepsilon) \cup (\gamma_1 \times \mathcal{I}^\varepsilon)$, where $\gamma_1 := \gamma - \gamma_0$. We note with $\Gamma_1^\varepsilon := \gamma_1 \times \mathcal{I}^\varepsilon$, $\Gamma_1^{\pm,m,\varepsilon} := \gamma_1 \times \mathcal{I}^{\pm,m,\varepsilon}$, with self-explanatory notation, and $\widehat{\Gamma}^\varepsilon := \Gamma_\pm^\varepsilon \cup \Gamma_1^{\pm,\varepsilon}$. The upper and lower plate-like domains $\Omega^{+,\varepsilon}$ and $\Omega^{-,\varepsilon}$ are called the *adherents*, while the intermediate plate-like domain is called the *adhesive*.

We consider a three-layer plate occupying the reference configuration $\overline{\Omega}^\varepsilon \times [0, T]$ at a positive time $T > 0$, see Fig. 1. We study the physical problem corresponding to the mechanical behavior of an anisotropic non homogeneous linearly elastic three-layer plate of thickness $2h^\varepsilon := 2h^{m,\varepsilon} + 2h^{+,\varepsilon} + 2h^{-,\varepsilon}$ and middle surface $\overline{\omega}$, with mass densities $\rho^{\pm,m,\varepsilon} > 0$. The sets $\Omega^{m,\varepsilon}$, $\Omega^{+,\varepsilon}$ and $\Omega^{-,\varepsilon}$ are filled by three anisotropic non homogeneous linearly elastic materials whose constitutive laws are defined as follows:

$$\sigma_{ij}^\varepsilon(\mathbf{u}^\varepsilon) = C_{ijk\ell}^\varepsilon e_{k\ell}^\varepsilon(\mathbf{u}^\varepsilon),$$