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Effective spring boundary conditions for a damaged interface between dissimilar media in three-dimensional case

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ABSTRACT

Elastic waves in the presence of a damaged interface between two dissimilar elastic media is investigated in the three-dimensional case. The damaged is modeled as a stochastic distribution of equally sized circular cracks which is transformed into a spring boundary condition. First the scattering by a single circular interface crack between two dissimilar half-spaces is investigated and solved explicitly for normally incident waves in the low frequency limit. The transmission by a distribution of cracks is then determined and is transformed into a spring boundary condition, where effective spring stiffnesses are expressed in terms of elastic moduli and damage parameters. A comparison with previous results for a periodic distribution of cracks shows good agreement.

1. Introduction

Elastic wave scattering by damage and delaminations is of considerable importance for ultrasonic non-destructive evaluation and structural health monitoring, where ultrasound is widely used to detect interfacial damage. Ultrasound methods should distinguish between an open crack, where all the faces are stress-free, and a delamination. A delamination can be more complex than an open crack: faces may interact or consist of multiple microcracks, especially at adhesive bonds. Identification of damaged interfaces or zones of nonperfect contact between materials is more complicated than identification of macrocracks. An imperfection can be simulated as a set or multiple cracks (Achenbach, 1989; Achenbach and Zhang, 1990) or as a deviation from perfect contact (Baik and Thompson, 1984; Tattersall, 1973), and this leads to a modification of the continuous boundary conditions at the interface. Although the approaches are technically different, they lead to similar results related to wave propagation in composites with damaged interfaces (Achenbach, 1989; Baik and Thompson, 1984; Golub and Boström, 2011; Kvasha et al., 2011).

It is natural to introduce a distribution of springs at the debonded interface (spring boundary conditions) in order to simulate it. Compared to multiple cracks, spring boundary conditions are simpler and more efficient. Thus, spring boundary conditions can be used for identification of multiple cracks (Shifrin, 2015). Tattersall (1973) showed experimentally that an imperfect contact can be investigated with elastic waves. This idea of using ultrasound in order to test adhesive bonds or debonded interfaces is applied by, e.g. Alers and Graham (1975), who demonstrated the applicability of spring boundary conditions for the estimation of adhesive bonds. The spring boundary conditions at the interface with normal $\boldsymbol{n}$ demand that the normal and tangential components of stress σ are continuous while the jump in the displacement vector is proportional to the stress:

$$\sigma^1_{ik} \cdot n_k = \sigma^2_{ik} \cdot n_k = \kappa_{ik}\left(u^2_k - u^1_k\right). \quad (1)$$

Here κ_{ik} is in general a three-by-three matrix and the upper indices number the contacting media.

Other approaches such as introducing a set of cracks or replacing a damaged layer by a thin layer have also been applied and compared with the spring model (Baik and Thompson, 1984; Sotiropoulos and Achenbach, 1988; Kachanov, 1994, etc.). Many studies exploiting distributed springs have been applied to simulate ultrasound interaction with planar damaged interfaces with different structures. Many of them (Baik and Thompson, 1984; Boström and Golub, 2009; Boström and Wickham, 1991; Lavrentyev and Rokhlin, 1994; Lekesiz et al., 2013a; Margetan et al., 1988; Pecorari, 2008) derived estimations for the distributed spring stiffnesses or applied these models in experimental work (Lavrentyev and Rokhlin, 1998; Leiderman and Castello, 2014). These derivations are often based on the idea of substitution of an array of planar cracks by distributed springs at the

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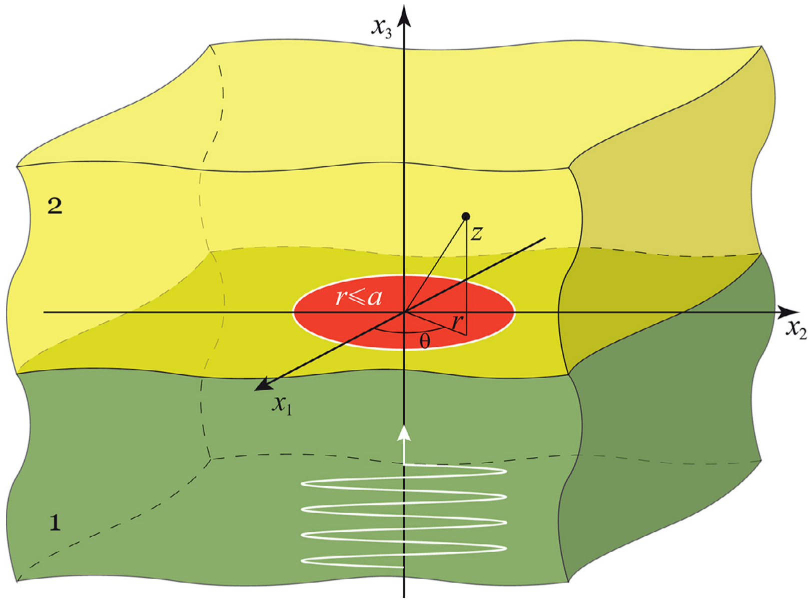

Fig. 1. Geometry of the problem for a single interface crack.

damaged interface. This substitution should lead to the same wavefields in the far-field zone from the interface. It should be mentioned that normal and transverse spring stiffnesses are equal in the case of in-plane motion (Golub and Boström, 2011; Lekesiz et al., 2011), while they differ in the three-dimensional case (Lekesiz et al., 2013a). Baik and Thompson (1984) used a quasi-static approximation for plane P-waves and obtained the expression for effective normal spring stiffness for identical materials. Margetan et al. (1988) extended this approach and estimated the transverse component of the spring stiffness for identical media, Lavrentyev and Rokhlin (1994) derived the stiffnesses for the case of dissimilar materials at the imperfect contact zone. An effective spring stiffness approximation was proposed for a planar periodic array of collinear cracks (Lekesiz et al., 2011) and a hexagonal array of coplanar penny shaped cracks located at the interface between two dissimilar solids (Lekesiz et al., 2013a).

The present paper is an extension of previous work on distributions of strip-like cracks between dissimilar media (Boström and Golub, 2009; Golub, 2010; Golub and Boström, 2011; Kvasha et al., 2011) and the study of Boström and Wickham (1991), where a distribution of circular contacts between two identical half-spaces were considered. The aim of this study is to obtain expressions for the spring boundary conditions in three dimensions describing wave propagation through a damaged interface between dissimilar isotropic media in terms of elastic moduli and damage parameters. First, an integral equation for a circular interface crack is derived following Krenk and Schmidt (1982) combined with an integral equation technique (Glushkov and Glushkova, 2001; Glushkov et al., 2002). The scheme used by Boström and Wickham (1991) is applied in order to obtain the total transmission coefficients for a distribution of cracks using a reciprocal theorem and ensemble averaging. In order to construct analytical formulae, an asymptotic low frequency solution for a single circular crack between dissimilar half-spaces is derived, see Ohyoshi (1973); Vatulyan and Yavruyan (2006). Then the reflection and transmission coefficients for normal incidence of a plane P-wave and S-wave for a random distribution of equally sized circular cracks at the interface between two half-spaces are calculated. The diagonal components of the spring stiffness matrix are derived from the equality of the transmission coefficients for the spring model and the damaged interface.

2. Single interface crack

In this section, the scattering of time harmonic waves by an open, circular crack at the interface between two dissimilar elastic isotropic half-spaces is investigated. Cartesian (x_1, x_2, x_3) and cylindrical (r, θ, z) coordinate systems are used in the following, both are centred at the circular crack occupying the domain $\Omega = \{r \le a, z = 0\}$ as depicted in Fig. 1. The displacement vector is denoted $\boldsymbol{u}^j = u_i^j = \{u_1^j, u_2^j, u_3^j\}$, where superscript $j = 1$ corresponds to the lower half-space ($x_3 < 0$) and $j = 2$ to the upper one ($x_3 > 0$). The material properties are determined by the Lamé constants λ_j and μ_j and densities ρ_j. The wave numbers $k_{ij} = \omega/v_{ij}$ at the angular frequency ω are expressed via the longitudinal or P wave velocity v_{1j} and the transverse or S wave velocity v_{2j}:

$$v_{nj} = \sqrt{c_{nj}/\rho_j}, \quad c_{1j} = \lambda_j + 2\mu_j, \quad c_{2j} = \mu_j.$$

Harmonic motion in isotropic media is governed by the Lamé equation

$$\sum_{i=1}^{3} \frac{\partial \sigma_{ik}^j}{\partial x_k} + \rho^j \omega^2 u_i^j = 0, \qquad j = 1, 2. \tag{2}$$

The stress tensor components are given by Hooke's law:

$$\sigma_{ik}^j = \lambda_j \left(\frac{\partial u_1^j}{\partial x_1} + \frac{\partial u_2^j}{\partial x_2} + \frac{\partial u_3^j}{\partial x_3} \right) \delta_{ik} + \mu_j \left(\frac{\partial u_i^j}{\partial x_k} + \frac{\partial u_k^j}{\partial x_i} \right).$$

where δ_{ik} is the Kronecker delta.

The total field in the two half-spaces with a circular interface crack is a sum of an incident field in the absence of the crack $\boldsymbol{u}^{\text{in}}$ and a scattered field $\boldsymbol{u}^{\text{sc}}$ due to the crack. For the purposes of this study the incident field $\boldsymbol{u}^{\text{in}}$ is taken as a plane wave propagating along the x_3 axis in the lower half-space plus the corresponding reflected and transmitted waves:

$$\boldsymbol{u}_s^{\text{in}}(x_1, x_2, x_3) = \begin{cases} \boldsymbol{p}_s \left(\mathrm{e}^{\mathrm{i}k_{1s}x_3} + R_s^- \mathrm{e}^{-\mathrm{i}k_{1s}x_3} \right), & x_3 < 0, \\ \boldsymbol{p}_s T_s^- \mathrm{e}^{\mathrm{i}k_{2s}x_3}, & x_3 > 0, \end{cases} \tag{3}$$

The index s here takes the value $s = 1$ for an incoming P wave and $s = 2$ for an incoming S wave. The amplitude reflection and transmission coefficients are