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Propagation of non-axisymmetric waves in an infinite soft electroactive hollow cylinder under uniform biasing fields

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Based on Dorfmann and Ogden's nonlinear theory of electroelasticity and the associated linear incremental theory, the non-axisymmetric wave propagation in an infinite incompressible soft electroactive hollow cylinder under biasing fields is investigated. The biasing fields are uniform, including an axial pre-stretch and a radial stretch in the plane perpendicular to the axis of the cylinder as well as an axial electric displacement. Such biasing fields make the originally isotropic electroactive material behave during its incremental motion like a conventional transversely isotropic piezoelectric material, hence greatly facilitating the following analysis. The three-dimensional equations of wave motion in cylindrical coordinates are derived and exactly solved by introducing three displacement functions. The exact solution is expressed in terms of Bessel functions, and explicit frequency equations are presented in different cases. For a prototype nonlinear model of electroactive material, numerical results are given and discussed. It is found that the initial biasing fields as well as the geometrical parameters of the hollow cylinder have significant influences on the wave propagation characteristics.

1. Introduction

Soft electroactive materials are smart materials, which may be produced by embedding electroactive particles in a rubber-like matrix such as silica gel and silicone rubber (Bossis et al., 2001). They have attracted considerable interests and are widely used to develop high-performance mechanical devices such as actuators and artificial muscles because of their rapid response and large deformation under electrical stimulus (Anderson et al., 2012; Henann et al., 2013).

Nonlinear analysis of soft electroactive materials or structures is quite complex due to the strong nonlinearity as well as the electromechanical coupling. The formulation of the general nonlinear theory of electroelasticity dates back to the 1950s. Toupin (1956, 1963) first established the theories governing the static and dynamic responses of elastic dielectrics. Tiersten (1971) later extended Toupin's study to the case with thermal effect. The nonlinear interactions between the mechanical and electromagnetic fields are well expounded in the books by Landau and Lifshitz (1960), Nelson (1979), and Maugin (1988), to name a few. Theoretical development of the nonlinear theories of electroelasticity has been revived in the recent decade (Dorfmann and Ogden, 2005, 2006; McMeeking and Landis, 2005; Mockensturm and Goulbourne, 2006; Bustamante et al., 2009; Suo, 2010) since new soft electroactive materials have been produced, indicating a very tempting prospect of applications.

The study on waves in electroactive materials not only presents significant theoretical interests but also is of specific practical importance. Chai and Wu (1996) applied the Lothe–Barnett's integral formalism to the study of surface waves in a prestressed piezoelectric material. The initial stress effect on the reflection coefficients of waves in a prestressed piezoelectric half-space was discussed in a recent paper by Singh (2010). Based on the nonlinear framework for electroelasticity (Dorfmann and & Ogden, 2005, 2006) and the associated linear incremental theory (Dorfmann and & Ogden, 2010b), Dorfmann and Ogden (2010a) analyzed the plane waves propagating in a homogeneously deformed electroactive material and the surface waves in a homogeneously deformed half-space of incompressible electroactive material. Axisymmetric waves in pre-stretched incompressible soft electroactive cylinders were examined in an exact manner by Chen and Dai (2012), also based on the theoretical framework suggested by Dorfmann and Ogden. In a more recent paper, Su and Chen (2014) extended Chen and Dai's work to a cylindrical shell and further considered the influence of the electric field exterior to the shell. Almost simultaneously, Shmuel et al. (2012)

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showed a strong effect of the biasing fields on the propagation of Rayleigh–Lamb waves in dielectric layers. Axisymmetric waves in dielectric elastomer tubes under biasing fields were also studied by Shmuel and deBotton (2013), where the biasing field is produced by applying a voltage difference between the inner and outer surfaces of the shell. This is quite different from that in Chen and Dai (2012) and Su and Chen (2014), which actually results in nonuniform biasing fields, and makes it impossible to obtain exact solutions.

In this paper, we aim at developing an exact analysis of non-axisymmetric waves in an infinite soft electroactive hollow cylinder subjected to uniform pre-stretch and/or biasing electric field. This is an extension of our previous works mentioned above, where only the simple axisymmetric case was considered. For the purpose of analysis, the theories of nonlinear electroelasticity and linear incremental field proposed by Dorfmann and Ogden (2005, 2006, 2010b) are briefly reviewed. As in Chen and Dai (2012), uniform biasing fields in cylindrical coordinates are assumed here to enable an exact analysis. The three-dimensional equations governing the small-amplitude non-axisymmetric waves in incompressible soft electroactive hollow cylinders under uniform biasing fields are simplified and decoupled by introducing three displacement potentials. An exact solution is then derived in terms of Bessel functions. Numerical examples are finally presented to show the effects of biasing fields and other parameters on the wave propagation behavior.

2. Basic formulations

2.1. Nonlinear theory of electroelasticity

Consider an incompressible continuous electroelastic body. We denote the undeformed, stress-free configuration by B_r, and its boundary by ∂B_r, with $\mathbf{N}$ being the outward unit normal. Any material particle, say X, is labeled by a position vector $\mathbf{X}$. Let B_t denote the corresponding deformed configuration with ∂B_t the boundary and $\mathbf{n}$ the outward unit normal. The deformation is described by the mapping $\mathbf{x} = \chi(\mathbf{X}, t)$ where $\boldsymbol{\chi}$ is a continuous and twice differentiable vector function. The deformation gradient is defined by $\mathbf{F} = \mathrm{Grad}\,\chi$ with the Cartesian components given by $F_{i\alpha} = \partial x_i/\partial X_\alpha$. $\mathbf{b} = \mathbf{F}\mathbf{F}^T$ and $\mathbf{c} = \mathbf{F}^T\mathbf{F}$ are the left and right Cauchy–Green tensors respectively. The relations between the infinitesimal undeformed surface element $\mathrm{d}A$ and volume element $\mathrm{d}V$ and those deformed ones are specified by $\mathbf{n}\mathrm{d}a = J\mathbf{F}^{-\mathrm{T}}\mathbf{N}\mathrm{d}A$ and $\mathrm{d}v = J\mathrm{d}V$ respectively, where $J = |\mathbf{F}|$ is the determinant of the deformation gradient $\mathbf{F}$, also known as the volume ratio. We have $J = 1$ for incompressible materials.

Under the ‘quasi-electrostatic approximation’, the appropriate specializations of Maxwell’s equations in the absence of free body charges and currents are

$$\mathrm{Curl}\,\mathbf{E}_l = \mathbf{0},\ \mathrm{Div}\,\mathbf{D}_l = 0, \tag{1}$$

where $\mathbf{E}_l = \mathbf{F}^T\mathbf{E}$ and $\mathbf{D}_l = \mathbf{F}^{-1}\mathbf{D}$ are the Lagrangian counterparts of the electric field vector $\mathbf{E}$ and electric displacement vector $\mathbf{D}$, respectively. Curl and Div are the curl and divergence operators defined in B_r, while curl and div will be used for the corresponding operators in B_t. The superscript T denotes the matrix transpose. In the vacuum outside the material, the electric field vector $\mathbf{E}*$ and electric displacement vector $\mathbf{D}*$ are related by

$$\mathbf{D}^* = \varepsilon_0\mathbf{E}^*, \tag{2}$$

where the constant ε_0 is the permittivity of vacuum. Obviously, we have

$$\mathrm{curl}\,\mathbf{E}^* = \mathbf{0},\ \mathrm{div}\,\mathbf{D}^* = 0. \tag{3}$$

The Maxwell stress in the vacuum is defined by

$$\tau^* = \varepsilon_0\left[\mathbf{E}^*\otimes\mathbf{E}^* - \frac{1}{2}(\mathbf{E}^*\cdot\mathbf{E}^*)\mathbf{I}\right]. \tag{4}$$

In the absence of surface charges, the jump conditions across the boundary read as

$$(\mathbf{E}-\mathbf{E}^*)\times\mathbf{n} = \mathbf{0},\quad (\mathbf{D}-\mathbf{D}^*)\cdot\mathbf{n} = 0. \tag{5}$$

The equations of equilibrium, in the absence of body forces, are

$$\mathrm{Div}\,\mathbf{T} = \mathbf{0}, \tag{6}$$

where $\mathbf{T} = \mathbf{F}^{-1}\tau = \partial\Omega/\partial\mathbf{F} - p\mathbf{F}^{-1}$ is the nominal stress tensor, with $\boldsymbol{\tau}$ being the total Cauchy stress tensor, $\Omega(\mathbf{F}, \mathbf{D}_l)$ is an amended energy function defined per unit volume in the reference configuration, and p is a Lagrange multiplier associated with the incompressibility constraint. p is identified as a hydrostatic pressure in Holzapfel (2000) and Dorfmann and Ogden (2014).

The mechanical boundary condition is given by

$$\tau\mathbf{n} = \mathbf{t}_a + \mathbf{t}_e, \tag{7}$$

here $\mathbf{t}_a$ is the applied mechanical traction per unit area of ∂B_t, and $\mathbf{t}_e = \tau^*\mathbf{n}$ is the contribution to the traction due to the electric field exterior to the body. Note that $\mathbf{t}_e$ is an unknown quantity, to be determined from the governing equations and the jump conditions.

2.2. Linear theory for incremental field

Following the formulation of Dorfmann and Ogden (2010b), we superimpose an incremental deformation $\dot{\mathbf{x}}(\mathbf{X}, t)$ along with an increment in the electric displacement $\dot{\mathbf{D}}_l$ upon the deformed configuration. The superposed dot is used in this paper to denote incremental quantities. The incremental forms of the governing Eqs. (1) and (6) are

$$\mathrm{curl}\,\dot{\mathbf{E}}_{l0} = \mathbf{0},\quad \mathrm{div}\,\dot{\mathbf{D}}_{l0} = 0, \tag{8}$$

$$\mathrm{div}\,\dot{\mathbf{T}}_0 = \rho\mathbf{u}_{,tt}, \tag{9}$$

where $\mathbf{u}(\mathbf{x}, t) = \mathbf{u}(\chi(\mathbf{X}, t), t) = \dot{\mathbf{x}}(\mathbf{X}, t)$ should be noticed, $\dot{\mathbf{T}}_0 = \mathbf{F}\dot{\mathbf{T}}$, $\dot{\mathbf{E}}_{l0} = \mathbf{F}^{-\mathrm{T}}\dot{\mathbf{E}}_l$, $\dot{\mathbf{D}}_{l0} = \mathbf{F}\dot{\mathbf{D}}_l$ are the ‘push forward’ versions of $\dot{\mathbf{T}}$, $\dot{\mathbf{E}}_l$, $\dot{\mathbf{D}}_l$ respectively. The linear incremental constitutive equations for an isotropic electroactive material are

$$\dot{\mathbf{T}}_0 = \mathbf{A}_0\mathbf{H} + \boldsymbol{\Gamma}_0\dot{\mathbf{D}}_{l0} + p\mathbf{H} - \dot{p}\mathbf{I},\quad \dot{\mathbf{E}}_{l0} = \boldsymbol{\Gamma}_0^T\mathbf{H} + \mathbf{K}_0\dot{\mathbf{D}}_{l0}, \tag{10}$$

where $\mathbf{H} = \mathrm{grad}\,\mathbf{u}$ is the displacement gradient. The components of the instantaneous electroelastic moduli tensors in Eq. (10) are

$$A_{0piqj} = F_{p\alpha}F_{q\beta}A_{\alpha i\beta j} = A_{0qjpi},\ \Gamma_{0piq} = F_{p\alpha}F_{\beta q}^{-1}\Gamma_{\alpha i\beta} = \Gamma_{0ipq},$$
$$K_{0ij} = F_{\alpha i}^{-1}F_{\beta j}^{-1}K_{\alpha\beta} = K_{0ji}, \tag{11}$$

with

$$A_{\alpha i\beta j} = \frac{\partial^2\Omega}{\partial F_{i\alpha}\partial F_{j\beta}},\quad \Gamma_{\alpha i\beta} = \frac{\partial^2\Omega}{\partial F_{i\alpha}\partial D_{l\beta}},\quad K_{\alpha\beta} = \frac{\partial^2\Omega}{\partial D_{l\alpha}\partial D_{l\beta}}. \tag{12}$$

Obviously, they depend on the applied biasing fields. Thus, the biasing fields can be a useful means to adjust the instantaneous material properties, which in turn have a profound effect on the incremental fields.

Similarly, the incremental forms of Maxwell’s equations outside the material are

$$\mathrm{curl}\,\dot{\mathbf{E}}^* = \mathbf{0},\quad \mathrm{div}\,\dot{\mathbf{D}}^* = 0. \tag{13}$$

The incremental fields $\dot{\mathbf{E}}^*$ and $\dot{\mathbf{D}}^*$ are related by $\dot{\mathbf{D}}^* = \varepsilon_0\dot{\mathbf{E}}^*$. Accordingly, the incremental forms of the boundary conditions (5) and (7) are

$$(\dot{\mathbf{E}}_{l0} - \dot{\mathbf{E}}^* - \mathbf{H}^{\mathrm{T}}\mathbf{E}^*)\times\mathbf{n} = \mathbf{0},\ (\dot{\mathbf{D}}_{l0} + \mathbf{H}\mathbf{D}^* - \dot{\mathbf{D}}^*)\cdot\mathbf{n} = 0, \tag{14}$$

$$\dot{\mathbf{T}}_0^{\mathrm{T}}\mathbf{n} = \dot{\mathbf{t}}_{A0} + \dot{\tau}^*\mathbf{n} - \tau^*\mathbf{H}^{\mathrm{T}}\mathbf{n}, \tag{15}$$

where $\dot{\mathbf{t}}_{A0}\mathrm{d}a = \dot{\mathbf{t}}_A\mathrm{d}A$, with $\mathbf{t}_A$ being the applied mechanical traction per unit area of ∂B_r, and $\dot{\tau}^*$ is the incremental Maxwell stress given by

$$\dot{\tau}^* = \varepsilon_0[\dot{\mathbf{E}}^*\otimes\mathbf{E}^* + \mathbf{E}^*\otimes\dot{\mathbf{E}}^* - (\mathbf{E}^*\cdot\dot{\mathbf{E}}^*)\mathbf{I}]. \tag{16}$$