International Journal of Solids and Structures 81 (2016) 274–293

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Limit analysis of transversally loaded masonry walls using an innovative macroscopic strength criterion

Gabriele Milani [a,*], Alberto Taliercio [b]

[a] Department of Architecture, Built Environment & Construction Engineering, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milan, Italy
[b] Department of Civil and Environmental Engineering, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milan, Italy

ARTICLE INFO

Article history:
Received 30 July 2015
Revised 2 November 2015
Available online 11 December 2015

Keywords:
Masonry
Homogenization
Transverse loads
Method of Cells
Limit analysis

ABSTRACT

The macroscopic strength properties of masonry walls with joints of finite thickness subjected to out-of-plane loads are estimated following an approach similar to the so-called Method of Cells for fiber-reinforced composites. A typical representative volume is subdivided into a few sub-cells, and a strain-rate periodic, piecewise differentiable transverse velocity field, depending on a limited number of degrees of freedom, is defined. Upper bounds to the macroscopic strength domain of the wall in the space of the macroscopic bending and twisting moments are obtained by applying the kinematic theorem of limit analysis within the framework of homogenization theory for periodic media. The approximated macroscopic failure surfaces are in good agreement with the 'exact' ones, available in the literature for infinitely strong units and infinitely thin joints, and with those obtainable by accurate 2D and 3D numerical models, at a much higher computational cost, for units of limited strength and joints of finite thickness. The influence of compressive in-plane loads and of the joint thickness on the macroscopic out-of-plane strength of the wall is also numerically investigated. Finally, the proposed model is applied to the prediction of the bearing capacity of laterally loaded masonry elements: the accuracy of the numerical predictions is assessed by comparisons with available experimental results and with more refined numerical models proposed by other authors.

1. Introduction

Masonry walls are extremely vulnerable to lateral loads, basically owing to the limited tensile strength of the joints. One of the most common failure modes observed for historical buildings after an earthquake is out-of-plane collapse of entire walls or parts of them: interesting overviews of typical earthquake-induced failure modes of ancient masonry buildings are reported e.g. by Spence and Coburn (1992), Giuffré (1993), D'Ayala and Speranza (2003) and De Felice (2011).

Taking the above comments into account, predicting the load bearing capacity of masonry walls subjected to transverse loads is of paramount importance to assess the safety of ancient buildings, including constructions of historical and monumental importance. In the past decades, several authors have addressed this topic both from an experimental and a theoretical point of view. Tests on masonry walls under out-of-plane loads were carried out e.g. by Southcombe et al. (1995). Simplified failure mechanisms were considered by Sinha (1978), De Felice and Giannini (2001) and Kelman and Spence (2003) to predict the bearing capacity of masonry structures subjected to lateral loads of various nature. Macroscopic strength criteria for masonry walls with a regular brick pattern were formulated by several authors (Sab, 2003; Milani et al., 2006; Sab et al., 2007; Cecchi et al, 2007; Dallot et al., 2008; Casolo and Milani, 2010; Milani, 2011) using the lower and upper bound theorems of limit analysis within the framework of homogenization theory for periodic media (Suquet, 1987). Some of the authors quoted above (Sab, 2003; Sab et al., 2007; Cecchi et al, 2007; Dallot et al., 2008) assume units to be rigid blocks separated by interfaces of vanishing thickness, so that failure and power dissipation are confined within the interfaces between adjacent blocks. In these papers, transversally loaded masonry walls are modeled as Kirchhoff–Love (thin) or Reissner–Mindlin (moderately thick) plates. Casolo and Milani (2013) derive the homogenized constitutive law of any masonry wall from the analysis of a typical Representative Volume Element (RVE) consisting of rigid units and elasto-plastic interfaces: these laws are implemented into a macroscopic model that discretizes masonry elements by rigid bodies connected by nonlinear springs, to carry out the pushover analysis of masonry structures. Milani et al. (2006) take the actual thickness of the joints into account, along with the possibility of failure within the units. A typical Representative Volume Element (RVE) of a running bond masonry

* Corresponding author. Tel.: +39 022399 4290; fax: +39 022399 4220.
E-mail address: gmilani@polimi.it, gabriele.milani77@gmail.com (G. Milani).

http://dx.doi.org/10.1016/j.ijsolstr.2015.12.004
0020-7683/

wall is subdivided into few rectangular elements, and polynomial expressions for the stress field of different order are proposed. The maximization problem that allows the macroscopic strength surface to be determined is solved numerically, and an important number of static minimization variables is involved.

Recently, Milani and Taliercio (2015) proposed an approach to define a macroscopic strength criterion for in-plane loaded periodic brickwork based on the upper bound theorem of limit analysis, in which any RVE is subdivided into six sub-cells, and simple collapse velocity fields, defined by a limited number of degrees of freedom and fulfilling suitable periodicity conditions, are considered. The actual thickness of the joints is taken into account, and both units and mortar joints are assumed to have finite strength. The approach stems from the so-called Method-of-Cells, originally proposed by Aboudi (1991) for unidirectional fiber reinforced composites, which has been successfully applied by Taliercio (2014) to propose closed-form expressions for the in-plane macroscopic elastic constants of periodic brickwork.

Following a similar approach, in this paper the macroscopic strength domain of masonry walls with a regular brick pattern and subjected to transverse loads is sought. Assuming any masonry wall to obey Kirchhoff–Love theory for thin plates, a transverse velocity field defined by six parameters only and corresponding to periodic microcurvatures is formulated over any RVE. The upper bound theorem of limit analysis is coupled to homogenization theory for periodic media (Suquet, 1987) to predict a macroscopic strength domain in the space of the bending and twisting moments. The bearing capacity of laterally loaded masonry elements is estimated neglecting the material heterogeneity and using the proposed homogenized strength criterion.

The layout of the paper is as follows. In Section 2 the velocity field proposed to estimate the homogenized strength domain of a RVE of brickwork is presented, separately considering the cases in which the RVE undergoes bending (Section 2.1) or twisting (Section 2.2). The kinematic approach employed to define the homogenized strength domain of masonry under out-of-plane loads is detailed in Section 3: solving a standard linear mathematical programming problem, several points of the approximated macroscopic failure surface are determined, each one representing an upper bound to the ultimate load bearing capacity of the wall under given moment combinations. In Section 4 numerical examples of homogenized strength domains in the space of the macroscopic moments are shown: their accuracy is assessed both by comparing the results obtained in case of thin joints and infinitely strong units with the exact solution available in the literature (Section 4.1), and by comparing the theoretical predictions with the results of numerical analyses carried out on 2D and 3D Finite Element models of any RVE (Section 4.2). The influence of a vertical compression on the macroscopic strength domain in the space of the bending and twisting moments is investigated in Section 5. In Section 6 the mathematical programming problem that allows the collapse load multiplier of a laterally loaded masonry structure to be computed using the upper bound theorem of limit analysis according to the proposed macroscopic strength criterion is outlined. In Section 7 the reliability of the predicted collapse multipliers and collapse mechanisms is assessed by comparing the theoretical predictions both with experimental results available in the literature and with the numerical results obtained by other authors. Transversally loaded masonry walls, either solid (Section 7.1) or windowed (Section 7.2), are considered. The influence of the joint thickness on the bearing capacity of the walls is also numerically investigated. Finally, in Section 8 the main findings of the work are summarized, together with the strong points of the proposed approach, and possible future developments are outlined.

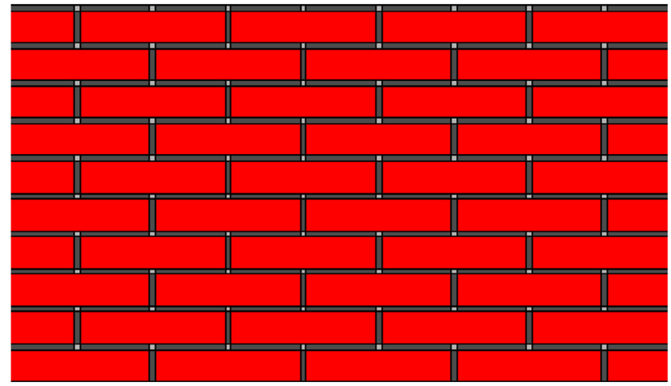

Fig. 1. Mid-plane of a masonry wall with regular brick pattern.

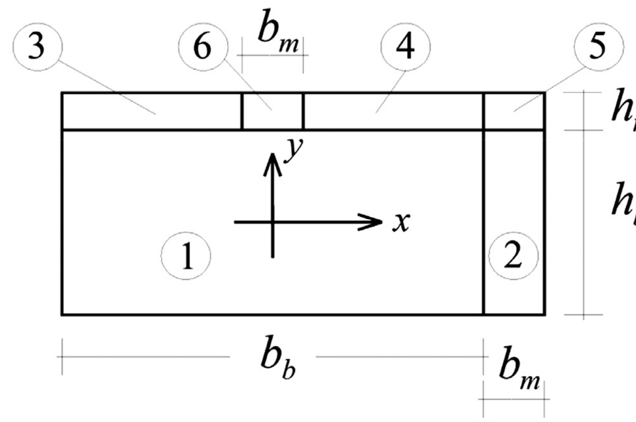

Fig. 2. 2D Representative Volume Element (RVE) and subdivision into sub-cells.

2. Microscopic velocity fields for a RVE of a transversally loaded masonry element

Consider a wall consisting of a regular pattern of bricks bonded by mortar joints of finite thickness. The mid-plane of the wall is assumed to be a plane of symmetry (Fig. 1). The homogenized (or macroscopic) mechanical properties of the wall can be deduced by analyzing a single RVE, also called 'unit cell' for periodic heterogeneous media. In Fig. 2 the intersection of the RVE with the mid-plane of the wall (Y) is shown.

Following an approach similar to the Method of Cells for fiber reinforced composites (Aboudi, 1991), the RVE is subdivided into a few sub-cells, and a polynomial displacement (or velocity) field is proposed within each sub-cell fulfilling suitable continuity and periodicity conditions. Fig. 2 shows the subdivision proposed for the RVE: sub-cell no. 1 corresponds to a brick, sub-cell no. 2 to a head joint, sub-cells 3 and 4 to a bed joint, and sub-cells 5 and 6 to cross-joints.

From here onwards, x denotes an axis parallel to the bed joints and y an axis parallel to the head joints. The origin of the local reference system Oxy is placed at the center of the brick.

In this work, attention is focused on the out-of-plane behavior of the wall. The wall is supposed to behave as a Kirchhoff–Love plate: the validity of this assumption, which was made in the past by several authors (Sab, 2003; Milani et al., 2006; Casolo and Milani, 2010; Milani, 2011), was checked by Dallot et al., (2008) through comparisons between theoretical and experimental failure loads for masonry plates and will be further assessed in Section 4.2 through comparisons with the numerical results of a 3D Finite Element model. According to this assumption, only the transverse velocity field, $w(x,y)$,