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Topology optimization and internal resonances in transverse vibrations of hyperelastic plates

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ABSTRACT

Nonlinear dynamics of plates synthesized using topology optimization and undergoing transverse vibrations with 1:2 internal resonances are presented. The plates are assumed to be made of hyperelastic materials; specifically two particular material models, namely, the neo-Hookean and Mooney–Rivlin material model are considered. A finite element approximation is first used in conjunction with novel topology optimization techniques to develop linearized candidate plate structures that have their lowest two natural frequencies in the ratio of 1:2. The plate structures are assumed to follow thin plate theory Kirchoff assumptions. The nonlinear dynamic response of the synthesized structures is then developed using modal superposition, and forced response to base excitations is analyzed to study the effects of material and geometric nonlinearities on nonlinear plate vibrations. The geometric nonlinearities introduced are through the assumptions of finite strains while the material nonlinearities arise due to nonlinear stress–strain or constitutive relationships for hyperelastic material models. Results are also compared with those obtained using von Karman and Novozhilov approximations for nonlinear plate vibrations. First the results are developed with the assumption that the materials are incompressible, and then this requirement is relaxed to include compressible materials as well.

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1. Introduction

The development of bio-mimetic sensors and actuators has attracted much attention in recent years. Concurrently and indeed, driven by the same goals, researchers have tried to use new materials, usually hyperelastic (Park et al., 2010) or electrostrictive (Pelrine et al., 2000), to build micro- and meso-scale devices. While generally not seen as frequently as beam-type elements in MEMS devices, plate structures are common in many such emerging applications including microjets (Oates and Liu, 2009) and energy-harvesting devices which can use ambient vibrations to produce electrical energy (Czech et al., 2010). References Pelrine et al. (2000) and Richards and Odegard (2010) describe several advantages of hyperelastic materials such as low cost, weight, ability to withstand large strains and ease of manufacturing in various shapes and configurations. Owing to such advantages, diverse applications such as micro-fluidic pumps (Xia et al., 2005), high-speed micro-actuators (Pelrine et al., 2000) and "soft" robotic systems (Petralia and Wood, 2010) have been fabricated using hyperelastic polymers.

Nonlinear response of elastic structures to resonant excitations has been investigated for quite some time, e.g. see Sathyamoorthy (1998) and Lacarbonara (2013). More specifically, "internal resonance" in a structure is possible when the natural frequencies of two or more modes are commensurable or nearly commensurable. In the presence of nonlinearities and a sufficient level of excitation, these frequency relations can lead to energy transfer between modes; for example if a system has quadratic nonlinearities and has two of its linear modal frequencies in the ratio of 1:2, it can exhibit 1:2 internal resonance if the excitation amplitude is above some threshold (Bajaj et al., 1994; Balachandran and Nayfeh, 1990; Wang and Bajaj, 2010). Internal resonance on its own has also been proposed as a mechanism for resonant MEMS based sensing (Vyas et al., 2009; 2008).

The mechanisms of generating quadratic nonlinearities in the structure can be roughly grouped under two categories, namely, geometric and material. Geometric nonlinearities can be introduced due to large deformations, non-flat equilibrium conditions (such as for a curved arch Tien et al., 1994) or structural asymmetry (Thomas et al., 2005). Material nonlinearities are caused by the structures having a non-linear stress–strain relationship, or in other words, the coupling between forcing and displacements is non-linear which might lead to situations where doubling of forcing amplitude may not lead to doubling of structural displacement. In this work, both geometric nonlinearities by virtue of large deformations, and material nonlinearities by virtue of a nonlinear hyperelastic constitutive law contribute to

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generation of quadratic nonlinearities in the structures to make 1:2 internal resonance possible.

The aim of this work is to present a methodology for design and analysis for nonlinear dynamic response of resonators incorporating hyperelastic materials and undergoing 1:2 internal resonance in large amplitude transverse vibrations. The resonators are essentially thin rectangular plates (with or without cutouts) made of isotropic hyperelastic materials. The resonance condition of having the lowest two natural frequencies close to the integer ratio of 2 is achieved by appropriate linear design of the structure obtained using a finite element analysis coupled to topology optimization. Once the linear design is complete, the mode shapes obtained from the linear finite element analysis can be used to construct a two-mode nonlinear model by expressing the displacements of the nonlinear structure in the 3-D space as a linear superposition of the two modes. This model is then used to develop the nonlinear dynamic response of the candidate structures and to analyze the occurrence of 1:2 internal resonances. Only a two-mode model is initially considered assuming that energy transfer occurs only between these two modes. More specifically, the second mode is directly excited using external resonant excitation in the transverse direction and it is hoped that it in turn excites the first mode due to the nonlinearities present in the structure.

Topology optimization has been an often used technique for structural optimization (Bendsoe and Sigmund, 2004) including optimization of resonators for their linear dynamic response (He et al., 2011). The application of topology optimization in the area of nonlinear dynamics is still in its infancy. This work explores the use of topology optimization techniques to synthesize structures for internal resonances by making a choice of an appropriate objective function. The use of hyperelastic material models such as the Mooney–Rivlin model and finite strains naturally introduce nonlinearities in the final equations of motion of these resonators. Topology optimization is done here using a linear finite element approximation making use of Kirchoff plate elements with the hyperelastic structure linearized around its flat equilibrium configuration (Finney and Kumar, 1988; Gruttmann and Taylor, 1992). The two specific topology optimization procedures considered in this work are the method of moving asymptotes (MMA) (Svanberg, 1987) and a simple iterative procedure introduced recently by the authors (Tripathi and Bajaj, 2014b). Both methods yield valid candidate structures. For constructing a more complete picture of the structure's deformation, it is important to have in-plane or membrane displacements independent of the transverse displacements which are not provided by the Kirchoff plate elements; hence, once the topology optimization has been completed, the final candidate structure is re-analyzed using four node shell elements to get independent in-plane deformation fields as well as to re-verify the satisfaction of resonance condition of the structure.

As for the nonlinear model development, while in general, the nonlinear response of a structure may consist of several modes, it is expected that in the presence of damping, modes with neither a direct excitation nor an internal resonance, will have their amplitudes diminish over time (Nayfeh, 2000). Therefore, a two-mode model may provide a fairly accurate representation of the system's nonlinear response. This two-mode approximation of displacements is then combined with the system Lagrangian (kinetic and potential energies) to develop a reduced-order model of the structure in which the principal unknowns are the two modal coefficients and their time derivatives. This Lagrangian along with Euler–Lagrange conditions gives the equations for the slow time evolution of amplitudes (Bajaj et al., 1994; Nayfeh, 2000) of the interacting modes. These slow time amplitude equations can be solved for obtaining the final nonlinear response of the structure.

Note that the system Lagrangian consists of kinetic energy of the velocity fields in the two modes and strain energies associated with the displacement fields. The strain energy forms for the hyperelastic structures depend specifically on the material constitutive laws, e.g., Mooney–Rivlin potential, and on the geometries of deformation, e.g., the von Karman, Novozhilov or some other strain measures associated with moderately thick or thin plate theories. Both these cases are considered in this work and the results are compared. The neo-Hookean material model is also considered for obtaining the nonlinear response of the system. Finally, some results are also presented for materials allowing for compressibility effects.

The paper is organized as follows: In Section 2, namely, linear structure synthesis, the topology optimization methods and the linear finite element formulation used to obtain candidate structures for internal resonance are described. A Matlab based finite element program is used with some results verified by the commercial software package ANSYS™ (ANSYS, 2014). In Section 3, the development of the nonlinear dynamic response of the structures obtained in Section 2 is presented. Different material models as well as geometry of deformation and effect of compressibility are considered in the context of the two-mode models. A case of higher mode models is also introduced and at least in the specific case, the higher modes are shown not to contribute to steady state response. Finally in Section 4, some conclusions are drawn along with discussion of the results.

2. Linear structure synthesis

The aim of the linear synthesis process is to obtain topologies of plates which can exhibit 1:2 internal resonances while undergoing nonlinear transverse vibrations. As mentioned earlier in the introduction, topology optimization techniques in conjunction with linear finite element analysis were used to obtain the candidate structures. For the purpose of optimization, the candidate structures were modeled with four-node thin-plate elements with three degrees of freedom per node (Cook et al., 2004). More specifically, the three degrees of freedom are the transverse displacement and the two rotations around the other two axes. The major purpose of the linear structure synthesis is to obtain a structure which has its lowest two natural frequencies in the ratio of 1:2. Note that the choice of lowest two natural frequencies is really no restriction as the same approach can be used for synthesis of the structure with any two chosen natural frequencies satisfying the desired condition. Analytically, this requirement can be specified by the relation:

$$\frac{\omega_2}{\omega_1} = 2, \tag{1}$$

where ω_1 and ω_2 are the first and the second natural frequency of the structure, respectively.

Based on this requirement, a topology optimization problem can be formulated whose solution would lead to structures which can exhibit 1:2 internal resonance in the nonlinear behavior. The objective function of this optimization problem can be stated as:

minimize

$$c(\omega) = \left(\xi - \frac{\omega_2}{\omega_1}\right)^2. \tag{2}$$

Note that this objective function does not limit the natural frequencies to a specific range, which may be desired in some applications. A constraint can be then imposed to specify the frequency range, thereby resulting in a constrained optimization problem. An example of such a case is the recent work of the present authors dealing with beam-like structures (Tripathi and Bajaj, 2014a). Also, it must be added that the minimization of the objective function given in Eq. (2) leads to an optimal structure which has its lowest two natural frequencies in the specified ratio 2 for 1:2 internal resonance. As mentioned in the previous section, the system needs to have quadratic nonlinearities and the frequency condition given by Eq. (1) to exhibit internal resonance. The linear synthesis method using topology optimization only provides the frequency ratio aspect of these requirements and not the existence of nonlinearity which, as

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