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Thermodynamically consistent elasto-plastic microplane formulation for fiber reinforced concrete

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ABSTRACT

In this work a thermodynamically consistent elasto-plastic microplane constitutive theory, aimed at simulating the failure behavior of Steel Fiber Reinforced Concrete (SFRC), is developed. The continuum (smeared crack) formulation, based on the microplane theory, assumes a parabolic maximum strength criterion in terms of normal and shear (micro-)stresses evaluated on each microplane to simulate the failure behavior of concrete. In the high confinement regime, a non-associated plastic flow rule is also defined in terms of microplane stresses. The well-known "Mixture Theory" is considered to account for the presence of fibers in concrete matrix. The interaction between steel fibers and cracked concrete in the form of fiber-to-concrete bond-slip and dowel mechanisms is taken into account. The complete formulation is fully consistent with the thermodynamic laws. After describing the proposed constitutive theory, numerical analyses at constitutive level of SFRC failure behavior are presented and discussed. Thereby, the variations of the fracture energy, post-peak strength and cracking behavior with the fiber contents are evaluated and compared against experimental data. The attention also focuses on the evaluation of the sensitivity of SFRC failure predictions with the proposed constitutive model regarding fiber orientation on one hand, and the bond-slip bridging actions and dowel mechanism on the other hand.

1. Introduction

The development of innovative composites based on further enhancing of cementitious materials represents a new challenging and interesting field of the Material Science and the Structural Engineering. Most significant examples are the High Performance Concretes (HPC) and, particularly, the Steel Fiber Reinforced Concrete (SFRC) (Gettu, 2008; Li et al., 1998a, 1998b; Mirsayah and Banthia, 2002). Actually, the application of SFRC in civil and military constructions have significantly increased in the last decades (and that trend still continues). The well-known deficiencies of cement-based materials like concretes, i.e., low strength and brittle response in low confinement and tensile regimes, can be mitigated by adding short steel fibers randomly distributed into the cementitious mortar. The major advantages of SFRC, as compared with plain concretes, is its higher residual tensile strength accompanied with elevated toughness in post-cracking regime (Naaman and Reinhardt, 2006; Nguyen et al., 2010; di Prisco et al., 2009). Since fiber bridging mechanisms mainly take place under cracked regime of concrete matrix, the mechanical behavior of uncracked members is practically not influenced by the addition of fibers beyond the limited increase of the elastic stiffness.

In the last years, many constitutive theories were proposed for failure analysis of SFRC. Most of them follow the Smeared Crack Approach (SCA) and, particularly, the flow theory of plasticity (Hu et al., 2003; Seow and Swaddiwudhipong, 2005) and the continuum damage theory, see also the work by Li and Li (2001). Besides the SCA-based proposals, several constitutive models and theoretical formulations are based on the Discrete Crack Approach (DCA). In the DCA the kinematic of cracking is modeled by means of the displacement field in discontinuities or interfaces in the finite element discretization, see also the contributions by Prasad and Krishnamoorthy (2002) and Etse et al. (2012).

The failure behavior of SFRC was evaluated not only at the macroscopic level of observation but also at the mesoscopic one. We may here refer to the contributions by Leite et al. (2004) and Schauffert and Cusatis (2012) who considered the effect of fibers dispersed into a Lattice Discrete Particle Model (LDPM), by Oliver et al. (2012) who highlighted the macroscopic response in terms

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of the meso-structural phenomenon associated with the fiber–matrix bond-slip action, by Gal and Kryvoruk (2011) who proposed a mesoscale two-step homogenization approach and the proposals by Radtke et al. (2010) and Cunha et al. (2012) whereby the SFRC has been considered as a two-phase material. A discrete crack model to predict failure behavior of SFRC based on “Mixture Theory” concepts allowing both macroscopic and mesoscopic analysis has been proposed by the authors (Caggiano et al., 2012; Etse et al., 2012).

During the last decades, the well-known microplane theory has largely been used for predicting the mechanical behavior of quasi-brittle materials such as concrete or rocks. Pioneer contributions of the microplane theory in constitutive formulations for concrete materials are represented by the works by Bažant and Gambarova (1984), Bažant and Oh (1985), Carol et al. (1992), and more recently by Carol and Bažant (1997), Kuhl and Ramm (2000) and Cervenka et al. (2005). A well-established thermodynamically consistent approach has been described by Carol et al. (2001) and Kuhl et al. (2001). Other relevant microplane-based contributions can be found in several applications including concrete failure prediction under cyclic loads (Ožbolt et al., 2001), numerical analyses of compressed concrete columns confined with carbon fiber reinforced polymers (Gambarelli et al., 2014), the mechanical response of polycrystalline shape memory alloys (Brocca et al., 2002), micropolar continua formulation in the spirit of Cosserat Media (Etse and Nieto, 2004; Etse et al., 2003), strain-softening nonlocal models (Bažant and Di Luzio, 2004; Di Luzio, 2007), large strains (Carol et al., 2004), as well as non-linear hardening–softening behavior of fiber reinforced concretes (Beghini et al., 2007; Caner et al., 2013). Although (Caner et al., 2013) these describe the behavior and fracturing of SFRC under not only uniaxial but also general multiaxial loading, they mainly include the fiber pull-out and breakage effects.

The present work formulates a novel thermodynamically consistent fracture-based microplane model for simulating the failure behavior of SFRC. The constitutive formulation at the microplane level is described in terms of normal and shear stresses vs. related micro-strains. Fiber effect on the composite failure behavior is taken into account through both a bond-slip formulation and a dowel model depending on the relative orientations between fibers and microplanes. The general basis of the proposed microplane theory for SFRC are presented in Section 2. Section 3 is related to the application of the well-known “Mixture Theory” by Truesdell and Toupin (1960) to describe the mechanical behavior of SFRC, following previous contributions by Oliver et al. (2008) and Vrech et al. (2010). Particularly, Section 3.1 reports the constitutive laws employed at microplane level featuring the fracture-based softening formulation for plain concrete, while the model description of the fiber-to-concrete interactions, approximating throughout debonding mechanisms and dowel effects of fibers crossing cracks, are highlighted in Sections 3.2–3.4. All constitutive laws of the different constituents are formulated within the thermodynamic framework. This encompasses both the elastic and inelastic portions of the internal material laws of the proposed constitutive theory. Finally, Section 4 covers the numerical analysis with the proposed constitutive model. The predictive capabilities and soundness of the proposal at constitutive level are addressed and discussed against experimental tests available in scientific literature. Section 4 also includes the numerical sensitive analysis of the model predictions regarding the variation of fiber direction, fiber content, bond-slip and dowel mechanisms.

2. Thermodynamically consistent microplane theory

A thermodynamically consistent elasto-plastic constitutive model based on the microplane theory is proposed for simulating the failure behavior of SFRC. Kinematic assumptions as well as constitutive equations are presented in the following subsections.

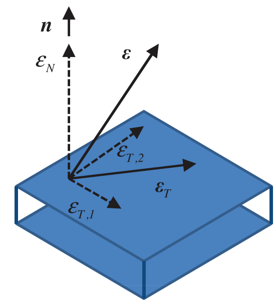

Fig. 1. Strain components at the microplane level.

The microplane approach originally proposed by Bažant and Oh (1983) consists in the formulation of constitutive laws at microplane level defining the mechanical behavior of planes (the microplanes) generically orientated. Then, the macroscopic response shall be achieved through the consideration of appropriated thermodynamically consistent homogenization process over the responses in all microplanes.

2.1. Kinematic assumptions

Assuming kinematic constraints, the normal and tangential strains at microplane level, ε_N and $\boldsymbol{\varepsilon}_T$, respectively, are computed by means of the following relationships:

$$\varepsilon_N = \boldsymbol{N} : \boldsymbol{\varepsilon}^{mac}, \quad \boldsymbol{\varepsilon}_T = \boldsymbol{T} : \boldsymbol{\varepsilon}^{mac} \tag{1}$$

being $\boldsymbol{\varepsilon}^{mac}$ the macroscopic strain tensor projected on a microplane characterized by its normal direction $\boldsymbol{n}$, see Fig. 1.

The projection tensors are defined as

$$\boldsymbol{N} = \boldsymbol{n} \otimes \boldsymbol{n}, \quad \boldsymbol{T} = \boldsymbol{n} \cdot \boldsymbol{I}^{sym} - \boldsymbol{n} \otimes \boldsymbol{n} \otimes \boldsymbol{n} \tag{2}$$

being $\boldsymbol{I}^{sym}$ the symmetric part of the fourth-order identity tensor.

In the elasto-plastic regime and assuming small strains, both macro- and microscopic strains are computed according to the Prandtl–Reuss additive decomposition. Particularly, at microplane level, normal and tangential strain rates are obtained as

$$\dot{\varepsilon}_N = \dot{\varepsilon}_N^e + \dot{\varepsilon}_N^p, \quad \dot{\boldsymbol{\varepsilon}}_T = \dot{\boldsymbol{\varepsilon}}_T^e + \dot{\boldsymbol{\varepsilon}}_T^p \tag{3}$$

where the supra-indexes e and p denote elastic and plastic components, respectively.

2.2. Thermodynamically consistent homogenization

Starting point for the formulation of thermodynamically consistent homogenization, relating the field variables on the microplanes with the macroscopic ones, is the definition of the macroscopic Clausius–Duhem inequality for isothermal processes as

$$\mathscr{D}^{mac} = \boldsymbol{\sigma}^{mac} : \dot{\boldsymbol{\varepsilon}}^{mac} - \dot{\psi}^{mac} \geq 0\,. \tag{4}$$

It sets that the macroscopic dissipation $\mathscr{D}^{mac}$, computed as the subtraction between the stress power, $\boldsymbol{\sigma}^{mac} : \dot{\boldsymbol{\varepsilon}}^{mac}$, and the evolution of the free-energy per unit mass of material $\dot{\psi}^{mac}$, cannot become negative; being $\boldsymbol{\sigma}^{mac}$ the macroscopic stress tensor.

Assuming the macro free-energy potential as the integral of the micro free-energy on a spherical region of unit volume Ω, the micro–macro free-energy relationship, according to Carol et al. (2001), can be expressed as

$$\psi^{mac} = \frac{3}{4\pi} \int_\Omega \psi \, d\Omega \tag{5}$$