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Topology optimization of hyperelastic structures with frictionless contact supports

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ABSTRACT

Hyperelastic structures usually undergo large deformations and thus may be subject to deformation-dependent contact supports. This paper presents an effective topology optimization methodology for the compliance-minimization design of hyperelastic structures with frictionless contact supports. In the optimization model, the strain-energy function of hyperelastic material is represented by an artificial penalization model, and the contact boundary conditions are modeled with hypothetical nonlinear springs. The additive hyperelasticity technique is employed for circumventing the local buckling instability exhibited by low-density elements. In conjunction with the adjoint variable sensitivity analysis, the nonlinear topology optimization problem is solved by a gradient-based mathematical programming algorithm. Numerical examples are given to show the importance of considering contact supports and to demonstrate the applicability of the proposed method.

1. Introduction

Hyperelastic materials, such as rubber-like materials and some polymers, have been applied in a wide variety of practical industrial applications. These materials usually exhibit a strongly nonlinear stress-strain relation and can experience very large reversible strains. This particular behavior of hyperelastic materials has attracted considerable interests in the study of accurate hyperelastic constitutive modeling (Martins et al., 2006) and structural design problems (Choi and Duan, 2000; Jarraya et al., 2014).

As a powerful tool for the conceptual design of products, topology optimization has become the most active research field in structural and multidisciplinary optimization during the past two decades (Sigmund and Maute, 2013; Deaton and Grandhi, 2014). For hyperelastic structures, there have been a number of works focusing on the optimal material distribution problem using topology optimization techniques incorporating the nonlinear finite element analysis. Bruns and Tortorelli (2001) investigated the topology optimization problem of hyperelastic structures and addressed the effects of geometrically and materially elastic nonlinearities on topology design. Lee and Youn (2004) developed a topology optimization method for the design of rubber vibration isolators. The hyperelastic constitutive model and the viscoelastic model are applied in the the static and dynamic analysis of rubber, respectively. Yoon and Kim (2007) studied the optimization problem of hyperelastic structures using an element connectivity parameterization method. Ha and Cho (2008) proposed a level set based topological shape optimization method for geometrically nonlinear hyperelastic structures. Klarbring and Strömberg (2013) studied the topology optimization of hyperelastic bodies subjected simultaneously to external forces and prescribed non-zero displacements. By adopting the total potential energy concept and the ground structure approach, Ramos and Paulino (2015) proposed a convex topology optimization method for the design of hyperelastic trusses. In particular, Labuerta et al. (2013) addressed the instability problem in the excessive distorted low density elements during the topology optimization process of large-deformation hyperelastic structures.

The above-mentioned works for topology optimization of hyperelastic structures were all performed under the assumption of fixed boundary conditions. Under a small strain assumption, these boundary conditions can be approximated as deformation-independent, considering the difference in the deformed and undeformed configurations are negligible. However, for a hyperelastic structure subjecting to large deformations, the boundary condition may change significantly.

As shown in Fig. 1(a), a hyperelastic beam is clamped at the left end, and a rigid circular support is placed at the right end. A vertical force F is applied to the middle point of the upper side. Under a very small force, the boundary condition at the right end can be simply modeled with a vertical support with fixed location, as shown in Fig. 1(b). However, as the force F increases and the beam further deflects, the support point on the rigid surface undergoes significant change, as shown in Fig. 1(c).

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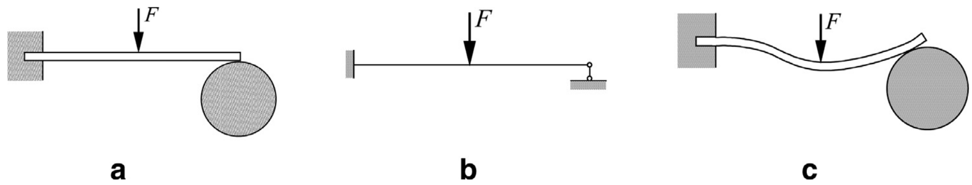

Fig. 1. A hyperelastic beam with frictionless contact supports. (a) Initial configuration. (b) Equivalent boundary condition under small deformation. (c) Deformed configuration under large deformation.

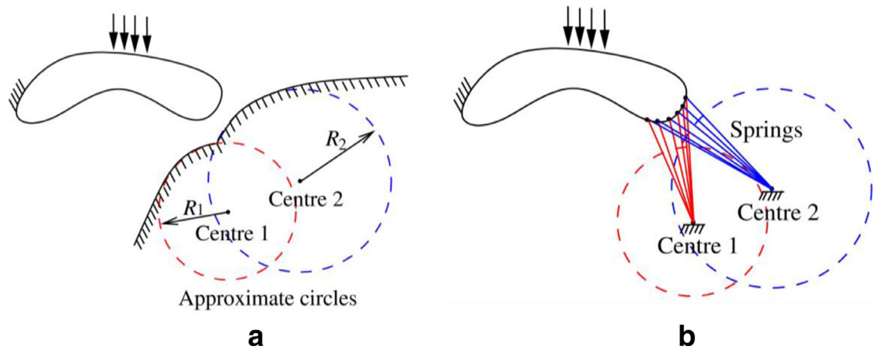

Fig. 2. Schematic representation of the nonlinear spring model. (a) Actual rigid support. (b) Nonlinear spring modeling.

This simple example illustrates how a large deformation of the hyperelastic structure can change the contact reaction of supports, which in turn affects the response of the structure. In practice, this deformation-dependent boundary may widely occur in MEMS devices (Mankame and Ananthasuresh, 2004), artificial joints and other biomedical applications (Yao et al., 1994).

During the last 10 years, the problem of topology optimization design involving contact has been extensively addressed in the literature by many researchers, mainly for linear elastic structures. Typically, Myśliński developed the topology and shape optimization of the elastic body in unilateral contact by the level set approach (Myśliński, 2008), the phase field approach (Myśliński, 2013) and the piecewise constant level set method (Myśliński, 2015). Based on the SIMP (solid isotropic microstructure with penalty) model, Fancello (2006) studied the stress-constrained topology optimization with contact boundary conditions. By using a smooth approximation of Signorini's contact conditions, Strömberg and Klarbring (2010) studied the topology optimization problem for linear elastic bodies with unilateral boundary conditions. Further, Strömberg (2012) proposed an optimization model of maximizing the potential energy, which will improve the numerical performance of this problem. In addition, Andrade-Campos et al. (2012) proposed a heuristic bone remodeling scheme to perform the 3D topology optimization problem with contact. Lawry and Maute (2015) investigated the topology design of two-phase material structures considering sliding contact and separation along interfaces. In these existing studies, contact problems are implemented mostly for the topology optimization of linear elastic structures. For the topology design of a hyperelastic body involving contact, numerical difficulties such as hyperelastic constitutive model and instability in low-density elements, will make the problem more complicated.

The aim of this work is to develop an effective topology optimization methodology for the design of hyperelastic structures with deformation-dependent contact supports. For simplicity, we assume the contact is frictionless and adhesionless. A nonlinear spring model is constructed to model the contact condition of a rigid surface. Using elemental density variables for the structural topology representation, an artificial material model with penalization for the strain-energy function is assumed. The additive hyperelasticity technique (Luo et al., 2015) is employed to avoid the instability phenomenon occurred in low-density elements during the topology optimization process. In this context, design sensitivities are obtained based on the adjoint variable scheme and the compliance-minimization problem is solved by using the Method of Moving Asymptotes (MMA) (Svanberg, 1987). Finally, numerical examples for topology optimization of large-deformation hyperelastic structures are presented to show the importance of considering contact supports and to demonstrate the applicability of the proposed method.

2. Nonlinear spring model for frictionless contact boundary

Consider a hyperelastic structure which is adjacent to a fixed rigid support as shown in Fig. 2(a). The potential contact surface of the rigid support is first approximated with several circular arc segments (spherical surface patches in 3D cases) with specified radiuses (R_1, R_2, ...). A flat surface can be naturally regarded as an arc segment with a sufficiently large radius. As the hyperelastic structure deforms under the action of external loads, contact at a point or an area occurs on the surface of the rigid support. This contact problem can be regarded as a displacement boundary condition that suddenly becomes active if the gap between the structure and the support surface closes. Note that the position and the reaction of contact points depend on the deformation of the hyperelastic structure.

In this study, a two-node nonlinear spring model, defined by a generalized force–displacement curve, is used to simulate the contact behavior of the frictionless rigid support. As shown in Fig. 2(b), the rigid support is replaced by a number of nonlinear springs whose stiffness varies from a zero value before contact to a large value after contact. After approximating the support boundary by circular arc segments, a group of springs are added between the fixed center of the approximate arcs and the potential contact points of the hyperelastic body. For each spring element, the internal force F_s will suddenly increase to a sufficiently large predefined value if the relative displacement of two nodes becomes less than a specified threshold value. Theoretically, using a step function for the force–displacement relationship of the spring element is able