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Shear properties of heterogeneous fluid-filled porous media with spherical inclusions

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ABSTRACT

An exact analytical solution is presented for the effective dynamic shear modulus in a heterogeneous fluid-filled poroelastic medium containing spherical inclusions. The complex and frequency-dependent properties of the derived shear modulus are solely caused by the physical mechanism of mesoscopic-scale wave-induced fluid flow whose scale is assumed to be smaller than wavelength but larger than the size of pores. Our model consists of three phases: a spherical inclusion, a shell of matrix material with different mechanical and/or hydraulic properties and an outer region of effective homogeneous medium of infinite extent. This three-phase model represents a self-consistent model or an approximate model of composite having periodically distributed inclusions. The behaviors of both the inclusion and the matrix are described by Biot's equations (1941) with standard conditions of Deresiewicz and Skalak (1963) at the inclusion-matrix interface. The effective medium is regarded as an equivalent elastic or viscoelastic material with complex and frequency-dependent moduli to be determined. The derived effective shear modulus is used to quantify the shear-wave attenuation and velocity dispersion. For the problem of fluid patchy saturation (inclusions with the same solid frame as the matrix but with a different pore fluid from that in the matrix), the gas pocket does not affect the shear attenuation and dispersion characteristic of the water-filled matrix medium at all. For the problem of double porosity structure (inclusions having a different solid frame than the matrix but the same pore fluid as the matrix), with the increase of frequency the heterogeneous medium transitions from a low-frequency state having drained inclusions and drained matrix with no pore pressure difference to a higher-frequency state having undrained inclusions and undrained matrix with no fluid communication at the inclusion's surface. The relaxation frequency at which the maximum value of inverse quality factor occurs moves to frequencies by two orders of magnitude lower if the size of a unit cell increases by one order of magnitude. Stiff inclusions imbedded in a relatively soft matrix can cause significant and observable attenuation at seismic frequency bands, but softer inclusions imbedded in a relatively stiff matrix cause very weak attenuation. The mixed heterogeneity in both the solid frame and pore fluid also has important influences on the frequency-dependent shear wave attenuation.

1. Introduction

The problems of velocity dispersion and attenuation of seismic waves in fluid-filled heterogeneous porous media have been of long-standing interest (e.g., Biot, 1956a,b; White, 1975; Dutta and Odé, 1979a,b; Berryman, 1985; Dvorkin, 1995; Pride and Berryman, 2003a; Müller et al., 2010; Song and Hu, 2013) with significant application in seismic exploration (e.g., Berryman et al. 2002; Ba et al., 2013) and acoustic well logging (e.g., Tang et al., 2012; Markova et al., 2014).

Earth crustal rocks are heterogeneous in various scales. Mesoscopic-scale wave-induced fluid flow (WIFF, Pride et al., 2004) whose scale is smaller than wavelength but larger than the size of pores generates heats and is believed to be the major physical mechanism in causing the intrinsic attenuation and velocity dispersion in seismic band of frequencies ($1-10^4$Hz) (Pride et al., 2004; Müller et al., 2010; Tisato and Quintal, 2013). From the body wave speed formula, it is well known that shear modulus not only critically determines the properties of the shear (S) wave but is significantly affects the properties of compressional (P) wave. Some

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experiments (Sams et al, 1997; Jones, 1986) show that the dispersion and attenuation of S waves could be of the same order of magnitude as those of P waves in fluid-saturated porous rocks. The present paper is devoted to study the effective shear properties in heterogeneous fluid-filled porous media by concerning mesoscopic WIFF.

Patchy saturation (inclusions having the same solid frame as the matrix but with a different pore fluid from the matrix) and double porosity (inclusions having a different solid frame than the matrix but the same pore fluid as the matrix) are two major models for seismic attenuation due to mesoscopic WIFF. Pride et al. (2004) showed that the heterogeneity in either the fluid or the solid frame can cause significant P-wave attenuation (maximum value of the inverse quality factor commonly lies in the range 0.01–0.1) in the seismic band of frequencies. To interpret the seismic attenuation and dispersion in heterogeneous porous rocks consisting of multiphase fluids and heterogeneous solids due to mesoscopic WIFF in porous rocks, many analytical solutions have been derived to quantify the dynamic bulk modulus and the attenuation in P waves (e.g., White, 1975; White et al, 1975; Dutta and Odé, 1979a,b; Berryman, 1985; Norris, 1993; Johnson, 2001; Pride and Berryman, 2003a; Pride et al., 2004; Müller and Gurevich, 2004; Brajanovski et al. 2005; Vogelaar and Smeulders, 2007; Vogelaar et al., 2010). For example, for consideration of energy loss in a patchy saturation reservoir rock, White (1975) derived the frequency-dependent effective bulk modulus by using a spherical gas-oil or gas-water pocket model. In his model, a small amount of gas in a fluid-saturated porous medium can cause significant velocity dispersion and attenuation in P waves (Dutta and Odé,1979a,b; Johnson, 2001; Pride et al., 2004; Vogelaar et al., 2010). Dutta and Odé (1979a,b) and Vogelaar et al. (2010) solved the problem exactly using the Biot consolidation theory of quasi-static poroelasticity and confirmed White's results. In addition, White (1975) suggested that the effective shear modulus is identical to the dry shear modulus of the porous composite because physically the fluid filling the pores changes neither the shear force existing in the solid frame nor the shear modulus. Pride and Berryman (2003a,b) developed double porosity theory to model acoustic propagation through heterogeneous porous structures with two different porous frames. Since the sphere is one of several typical and simple geometric inclusions, and is also a special case of the ellipsoid, it has been used to interpret the intrinsic P-wave attenuation in porous fluid-filled rocks. In addition to White's spherical-inclusion patchy model (White, 1975), Ba et al. (2011) considered a double-porosity medium consisting of spherical inclusions imbedded in an unbounded matrix having different porosity, permeability and compressibility. They derived the equations of motion showing that the spherical inclusion can cause significant fast P wave (which is similar to the ordinary P wave in elastic medium but has intrinsic energy loss in the propagation process) attenuation and dispersion in the seismic frequency band.

However, unlike the case of effective bulk modulus, deriving the effective shear modulus is rather complicated as it involves solving the shear deformation of phases. Recently, several papers have considered the shear properties in poroelastic media. For example, Berryman (2004) studied the dependence of macroscopic shear modulus on pore-fluid properties by using a locally isotropic layered model. He gave upper and lower bounds under drained and undrained boundary conditions, respectively. Masson and Pride (2007) estimated by finite difference modeling the shear attenuation and shear modulus dispersion in a double porosity model in which a soft porous ellipsoid is embedded within a stiffer matrix. They showed that significant attenuation over the seismic frequency range occurs as for P waves but smaller in magnitude. Quintal et al. (2012) used the finite-element method to model the S-wave modulus and attenuation in a 2-dimensional patchy saturation structure. They showed that the role of S-wave attenuation could be used as an indicator of fluid content in a reservoir in addition to P-wave attenuation. Other numerical methods studying the attenuation in shear waves can be found in Rubino et al. (2008), Wenzlau et al. (2010), Quintal et al. (2011) and Masson and Pride (2014). Liu et al. (2009) derived a series expansion solution for the scattering of incident S waves by mesoscopic-scale spherical inclusions embedded in an infinite poroelastic matrix. After obtaining the series expansion solution, they used a dilute model to calculate the scattering S-wave dispersion and attenuation. Significant S-wave attenuation is caused in the double porosity model at an S-wave wavelength that is approximately equal to the characteristic size of the inclusion. Although these papers above focus on the S-wave attenuation due to mesoscopic WIFF, unfortunately, to the best of our knowledge, no analytical solution for dynamic effective shear modulus in any typical structure has been given to account for the role of the mechanism of mesoscopic WIFF.

It is worth mentioning that the scattering approach is indeed a powerful tool to study the attenuation and velocity dispersion in heterogeneous porous media. As pointed out by Müller and Gurevich (2005), the processes of wave conversion scattering into Biot's slow P waves is equivalent to the mechanism of pore pressure relaxation due to wave-induced perturbation and thus also describes the mechanism of WIFF. An important application of the scattered Biot's slow P wave is to analyze the effective properties, attenuation and velocity dispersion of elastic waves in poroelastic materials. Berryman (1985) first analyzed the scattering of incident P wave by a porous spherical inclusion in an otherwise homogeneous porous matrix medium. One major difficulty is that the solution is given in terms of an infinite series, and one needs to solve a 6 × 6 system of equations for the coefficients at each harmonic. The analytical solution can be obtained only after some simplifications (e.g., Berryman, 1985; Ciz and Gurevich; 2005). Berryman (1985) obtained the asymptotic long-wavelength solution for the slow wave amplitude, which is valid for inclusions much smaller than the wavelength of Biot's slow wave. Ciz and Gurevich (2005) analyzed the amplitude of the Biot's slow wave for the scattering of incident P waves by spherical inclusions in the low-frequency variant of Biot's theory, and for the inclusion size much smaller than the wavelength of the fast P wave. A simple expression for the amplitude of the scattered Biot's slow wave was obtained by assuming that the amplitude of the scattered fast compressional wave is well approximated by the solution of the equivalent elastic problem. Ciz et al. (2006) applied Waterman–Truell multiple scattering theory to relate the scattering of a single inclusion to the wave attenuation in the poroelastic medium. Gurevich et al. (1998) applied the Born approximation to study the scattering of a fast P wave in a poroelastic medium by an ellipsoidal inclusion. The Born approximation is valid for low contrast of the inclusion's properties with respect to the matrix medium.

This paper focuses on dynamic effective shear modulus. The limitation of this paper is that the size of a unit cell shown in Fig. 1 is smaller than S-wave wavelength and much larger than the size of pores. Biot's equations for consolidation (Biot, 1941) of poroelastic media are employed to study the effective shear modulus for a heterogeneous medium containing spherical inclusions. The dispersive shear modulus is due to the heterogeneous distribution of pore fluid pressure and pore pressure diffusion. Furthermore, because no inertia terms appear in the equations for consolidation, the attenuation is solely due to flow of the viscous fluid, which is controlled by the gradient of the pore-fluid pressure. Excluding the inertial forces at low seismic frequencies is a valid approximation because they are negligible for typical properties of rocks and saturating fluids at these frequencies (e.g., Bourbie et al., 1987). The present work presents an analytical solution for the dynamic shear modulus of a heterogeneous poroelastic material containing