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A class of morphing shell structures satisfying clamped boundary conditions

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ABSTRACT

Many examples of multi-stable shell structures have been recently proposed with the underlying hypothesis of the shell being completely free on its boundary. We describe a class of shallow shells which are bistable after one of their sides is completely clamped. This result, which has relevant technological implications, is achieved by a suitable design of the initial, stress-free, shape.

1. Introduction

Engineering structures should often be able to face several load and operating conditions; their actual shape usually stems from a compromise, after a selection of the more challenging requirements. Instead, morphing structures optimize their response to external inputs by updating their geometric configuration; despite being quite recently introduced, they could become a standard in some areas of structural engineering (as foreseen in Daynes et al., 2011).

Here we focus on multistable shells, a particular example of morphing structures able to provide stiffness and strength whilst allowing considerable shape change. By triggering instability phenomena, or simply by exploiting displacement amplifications due to geometric nonlinearities, suitably designed shells could undergo major changes in shape under limited actuation forces.

For plates and shells, multistability can be achieved through a combination of means including pre-stresses, initial curvatures and plastic deformations. Indeed, a competition between geometric nonlinearities and elastic properties manages the accumulation and release of elastic energy in the deformation processes and determines the equilibrium configurations, their shapes and their stability. Since stable configurations can be quite different in their geometry and the transition between them may be accomplished by different load paths, the design of multistable shells calls for mathematical models and numerical tools able to depict a *global* stability scenario., i.e., capable of providing reliable information about the number and type of stable equilibria, the energy barriers interposed between them and the most appropriate actuation strategy (for instance, one demanding a preset amount of power to the actuators). This requires simplified shell models with few degrees of freedom, so as to obtain manageable solutions and perform qualitative analysis and quick parametric studies.

For shallow shells, which are the ones typically employed in technological applications, this task is achieved by reducing the Föppl–von Kármán (FvK) shell model (Föppl, 1907; Kármán, 1910) to a low-dimensional subspace ensuring a good approximation of the multiwell elastic energy (Seffen, 2007; Vidoli, 2013; Vidoli and Maurini, 2008). Specifically, we use the reduction procedure introduced in Vidoli (2013) to infer a three-degrees-of-freedom reduced model capable of predicting the multistable behavior of suitably curved cantilever shells.

Although the reduction strategy may be applied in quite general cases, for sake of simplicity we have chosen to focus our attention on shells with rectangular planforms where only one side is actually clamped. It has to be noted that, even if boundary conditions have to be taken into account when considering a bistable shell as a component of a complex structural system, only few works deal with the design of multistable constrained shells (Mattioni et al., 2009; Panesar and Weaver, 2012). Indeed, many literature studies were limited to examine the case of shells completely free on their

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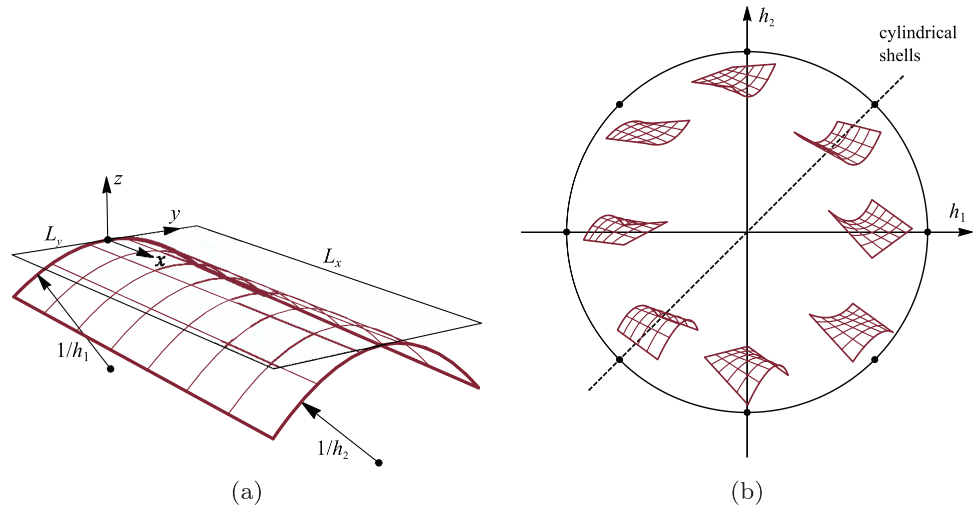

Fig. 1. (a) Design parameters for the natural stress-free configuration. (b) Resulting configurations in the curvature plane (h_1, h_2) for $L_x = L_y$.

sides, see for instance Norman et al. (2008), Pirrera et al. (2012), Coburn et al. (2013) and Lamacchia et al. (2015). Instead, we are able to predict, and numerically validate, several new examples of shells that are bistable after the application of the clamp constraint. These examples are characterized by combination of geometric and material parameters within a wide range of values, thus opening the way to a proper and more general approach to design and optimization of multistable shells.

The paper is organized as follows. In Section 2 we introduce the class of pseudo-conical shells. These are shallow shells, with rectangular planform, characterized by distinct curvatures, say h_1 and h_2, of two opposite sides; cylindrical shells are included as the special case $h_1 = h_2$. This class is sufficiently large to allow some room for optimization and sufficiently simple to allow a physical insight of the bistable response. In Section 3 the displacement ansatz to obtain the reduced shell model is introduced and discussed: the hypothesis of uniform curvature, used in several literature studies, must be abandoned in order to allow more complex shell configurations. Section 4 is devoted to show the main results: in particular, we give the conditions on the design parameters for a pseudo-conical shell to be mono- or bi-stable after clamping. For shells that are bistable, aimed at providing an effective actuation of such morphing structures, we provide a method to compute an optimal path between the two equilibria. In the same section, the predictions obtained by applying the reduced 3-dofs model are confirmed by comparison with refined FE simulations. Finally in Section 5 we draw some conclusions and discuss possible directions for future research.

2. Design parameters: natural shape and material constants

In this study a suitable choice of the natural (stress-free) configuration of the shell is sought as the primary mean to induce bistability; we do not consider here another well-known source of multistable behaviors, namely the presence of inelastic pre-stresses, see Hamouche et al..

While several authors already investigated natural configurations with uniform curvatures (see e.g. Seffen, 2007; Vidoli and Maurini, 2008; Fernandes et al., 2010; Coburn et al., 2013), we abandon this simplifying hypothesis to consider more general configurations. Specifically, within the shallow shell assumption, we restrict our attention to shells with pseudo-conic natural configurations. These last can be mathematically described by surfaces in the form:

$$\mathcal{S}_0 = \{(x, y, w_0(x, y)),\ 0 \le x \le L_x,\ -L_y/2 \le y \le L_y/2\}, \tag{1}$$

with

$$w_0(x, y) = \frac{y^2}{2}\left(h_1 + (h_2 - h_1)\frac{x}{L_x}\right), \tag{2}$$

for some $h_1, h_2 \in IR$ and $0 < L_y \le L_x$. Fig. 1 shows the meaning of these parameters and some of these shapes when the parameters h_1 and h_2 are varied for $L_x = L_y$.

The curvatures of the natural configuration are therefore not uniform and are given by:

$$h_x = 0, \quad h_{xy} = \frac{(h_2 - h_1)\,y}{L_x}, \quad h_y = h_1 + (h_2 - h_1)\frac{x}{L_x}. \tag{3}$$

Incidentally, cylindrical shells are included as the special case $h_2 = h_1$.

Three design parameters completely identify the natural shape of the shell: the aspect ratio $\eta = L_x/L_y \ge 1$ and the curvatures h_1 and h_2 which can be interpreted, cfr. Fig. 1a, as the curvatures in direction y of the sides $x = 0$ and $x = L_x$, respectively. As will be clear in the following, the area L_xL_y will play a role only in the scaling of curvatures based on the characteristic radius R, cfr. (25).

Concerning the constitutive properties of the material, for sake of simplicity we consider only homogeneous orthotropic shells with no coupling between bending and stretching (see Vannucci and Verchery, 2001). Specifically, assuming the principal material directions aligned with the coordinate directions x and y, the constitutive relations between bending moments M and curvatures k will read

$$M_x = D_{11}(k_x - h_x) + D_{12}(k_y - h_y), \quad M_{xy} = D_{33}(k_{xy} - h_{xy}),$$
$$M_y = D_{12}(k_x - h_x) + D_{22}(k_y - h_y), \tag{4}$$

while the relations between membranal stresses N and membranal strains ε will read:

$$N_x = A_{11}(\varepsilon_x - f_x) + A_{12}(\varepsilon_y - f_y), \quad N_{xy} = A_{33}(\varepsilon_{xy} - f_{xy}),$$
$$N_y = A_{12}(\varepsilon_x - f_x) + A_{22}(\varepsilon_y - f_y), \tag{5}$$

where strains $\{f_x, f_y, 2f_{xy}\}$ represent non-zero membrane stresses in the flat reference configuration, whilst curvatures $\{h_x, h_y, 2h_{xy}\}$ provide non-zero bending moments in the reference configuration. Moreover, since we do not consider inelastic pre-stresses, the Gauss compatibility equation holds true:

$$f_{x,yy} + f_{y,xx} - 2f_{xy,xy} = h_y h_x - h_{xy}^2$$

where $(\cdot)_{,x} = \partial(\cdot)/\partial x$, $(\cdot)_{,y} = \partial(\cdot)/\partial y$. Note that the bending moments vanish when the curvatures equal the ones in the natural configuration; similarly the membranal stresses vanish when