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Optimal structural arrangements of multilayer helical assemblies

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ABSTRACT

We report a quantitative framework to guide the braiding pattern design of multilayer helical assemblies. We optimize the structural pattern so as to maximize the construction's resistance to axial loads and concurrently minimize its torsional propensity. To that extent, we consider helical assemblies comprised of up to five layers, for which we identify favorable structural patterns, providing a database that covers most practical applications.

1. Introduction

Helical assemblies are load carrying structures with applications ranging from ropes and electricity power transfer cables to tissue engineering scaffolds (Papailiou 1997; Laurent et al., 2014). While a large number of studies has been devoted to the analysis of their mechanical properties, the selection of the structural arrangement itself has largely been disregarded, despite its rather detrimental impact on both the operational mechanical response and the structure's long term performance. Whereas an experimental retrieval of optimal structural patterns is infeasible, numerical simulations provide an ideal test-bed for this purpose.

A thorough description of the geometric properties of single, double and triple helical bodies has been provided by Lee (Lee, 1991), an analysis of primal use for the understanding of the structuring of helical assemblies. Helical assemblies are commonly encountered as substructures of larger constructions. Cable-bridge structures are characteristic examples of this kind, for which the cable tension level and placement controls the response of the overall construction (Janjic et al., 2003). Furthermore, helical assemblies are used in electric power transfer, with their design playing a crucial role in the minimization of power transfer losses (Sullivan, 1999). Their extensive use necessitated the characterization of their mechanical response, as the analytical and numerical modeling schemes bibliography indicates, primarily in the context of engineering cables.

Using analytical modeling, Lanteigne provided closed-formed solutions for the quantification of the mechanical response of helically armored cables upon axial, torsional and bending loads (Lanteigne, 1985). Accordingly, Raoof et al. developed simplified expressions for the stiffness coefficients of locked-coil strands (Raoof and Kraincanic, 1998), while Utting and Jones provided a large set of experimental data on single and three layer strands accompanied by closed-form stiffness expressions (Utting and Jones 1987a, 1987b). Furthermore, Costello conducted extensive experimental studies to characterize the mechanical response of wire ropes, complementing the experimental analysis with analytical, closed-form expressions for the structural response (Costello, 1990). Moreover, Sathikh et al. elaborated stiffness matrix coefficients for the axial and torsional strain response of helical bodies that hold symmetry considerations of the stiffness matrix (Sathikh et al., 1996), while Karathanasopoulos et al. extended the modeling approach to account for the effect of radial strain with contributions arising from the axial, torsional and bending helix cross section stiffness taken into account (Karathanasopoulos and Kress, 2015). Finally, the mechanical response of double-helix multi strand constructions to axial and torsional loads was analyzed, under the assumption that their constituents follow a fiber type response (Elata et al. 2004; Usabiaga and Pagalday 2008).

On the numerical modeling side, Jiang et al. estimated the structural properties of two layer strands using a reduced computational model that took advantage of the structural and loading symmetry (Jiang and Henshall 2000). Similarly, Stanova et al. worked on the axial stiffness properties of three layered strands (Stanova et al., 2011). A study on large spiral cables axial load-strain curves and failure loads was provided by Judge et al., the analysis based on three dimensional finite element modeling (Judge et al., 2012).

More recently, helical assembly applications that go beyond the context of engineering strands have come to the fore. In particular,

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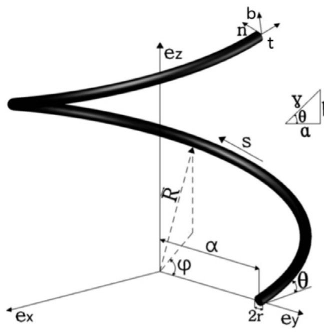

Fig. 1. Helical body geometry.

in the field of biomechanical engineering, helically braided scaffolds have been used for the restoration of tendon and ligament tissue (Laurent et al., 2014). Moreover, the development of artificial and biological material based applications such as nanotube helical ropes, has asked for a deeper understanding of their mechanical response, with bottom-up structural response models appearing in the literature (Zhao et al., 2014).

A significant number of studies have been devoted to assess the impact of loading bounds on the endurance and long-term functionality of helical assemblies. Argatov worked on the effect of inter-wire contact deformation of single layer rope strands making use of asymptotic modeling (Argatov, 2011). Alani et al. studied the correlation between the mean axial loading and the endurance limits of helical assemblies, to point out substantial variations associated with the helix angle selection of the individual layers (Alani and Raoof, 1997). Giglio et al. (Giglio and Manes, 2005) derived a linkage between the fatigue life and the stress state of ropes that are subject to axial and bending loads, suggesting that their bounds are directly related to fretting damage phenomena (Hobbs and Raoof, 1994). Finally, Chaplin performed a number of experimental studies that quantified the effect of different loading patterns on the life endurance of spiral ropes, illustrating the role of torsional loads as a failure mechanism (Chaplin, 2008).

The current work is structured as follows: We describe the engineering of a broad spectrum of helical assembly constructions comprised of up to five layers (Section 2). Amongst all possible structures, we identify torsionally counterbalanced arrangements of high axial stiffness for two, three, four and five layer constructions (Section 3, Appendix A). We comment on the retrieved optimal braiding patterns and conclude in Section 4.

2. Helical assembly modeling and optimization methodology

2.1. Helix geometry

The geometry of the helical assembly is characterized by the individual geometric properties of its constituents. A helix can be described through the following equation, formed with the use of the *Serret–Frenet* basis:

$$\mathbf{X}(x_n, x_b, s) = \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \mathbf{R}(s) + x_n\mathbf{n} + x_b\mathbf{b}, \quad -r \le x_n, x_b \le r \tag{1}$$

where r denotes the radius of the helix cross section and $\mathbf{R}(s)$ the centerline position vector of the helical body defined as follows:

$$\mathbf{R}(s) = \begin{Bmatrix} \alpha\cos\varphi \\ \alpha\sin\varphi \\ b\varphi \end{Bmatrix}, \quad \varphi = \frac{s}{\gamma}, \quad \gamma = \sqrt{\alpha^2 + b^2},$$
$$b = \alpha\tan\theta, \quad h = 2\pi b \tag{2}$$

In Eq. 2, a stands for the helix centerline position and b for the rise along the central axis of the helix per unit angular evolution φ upon which the helix height h for a period evolution is computed. The *Serret–Frenet* local base vectors are defined as follows:

$$\mathbf{n} = \begin{bmatrix} -\cos\varphi \\ -\sin\varphi \\ 0 \end{bmatrix}, \quad \mathbf{b} = \frac{1}{\gamma}\begin{bmatrix} b\sin\varphi \\ -b\cos\varphi \\ \alpha \end{bmatrix}, \quad \mathbf{t} = \frac{1}{\gamma}\begin{bmatrix} -\alpha\sin\varphi \\ \alpha\cos\varphi \\ b \end{bmatrix} \tag{3}$$

Fig. 1 schematically depicts the introduced parametrization.

2.2. Multilayer helical assembly parameter search space

We subsequently define the parameter space of the helical assemblies. For each helical layer, the cross section radius of the individual helical bodies r_i is allowed to vary by a maximum of 50% with respect to the radius of the core of the structure r_c, thus $0.5 \le r_i/r_c \le 1.5$. The layer centerline position of each layer i, named as a_i is defined as a function of the radius of all helical bodies in the different layers j, $\{r_j\}_{j=1}^{i}$ and of the core radius r_c, as schematically illustrated in Fig. 2. The helix angle of each layer θ_i is accordingly considered to vary within [70° 85°]. The angle selection allows for the constituents of the assembly to be primarily subject to normal rather than shearing stresses, while it guaranties a high axial strength for the overall construction. Furthermore, we allow for different layer orientation

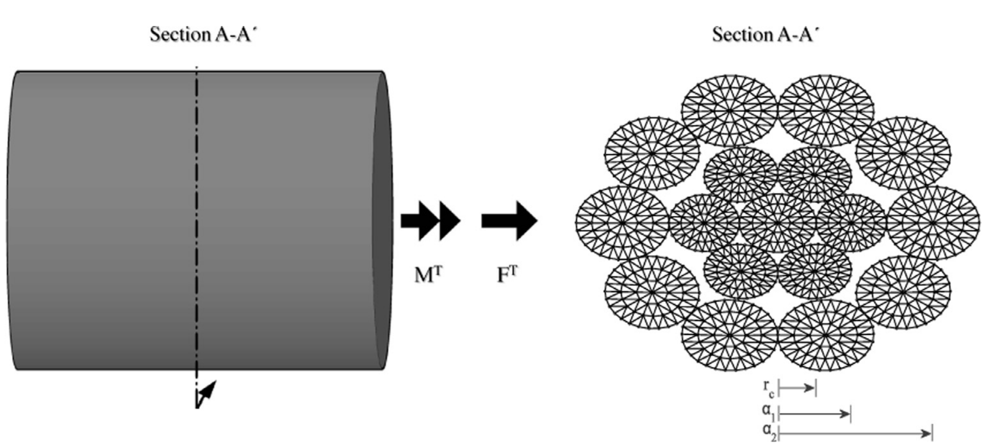

Fig. 2. Multilayer helical assembly geometry.