International Journal of Solids and Structures 78-79 (2016) 77–100

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Second-order two-scale asymptotic analysis for axisymmetric and spherical symmetric structure with periodic configurations

Qiang Ma [a,b], Junzhi Cui [c], Zhihui Li [a,b,*]

[a] Hypervelocity Aerodynamics Institute, China Aerodynamics Research and Development Center, Mianyang 621000, China
[b] National Laboratory for Computational Fluid Dynamics, BUAA, Beijing 100191, China
[c] LSEC, ICMSEC, Academy of Mathematics and Systems Science, CAS, Beijing 100190, China.

ARTICLE INFO

Article history:
Received 20 May 2015
Revised 27 August 2015
Available online 17 October 2015

Keywords:
Axisymmetric and spherical symmetric structure
Linear elastic problem
SOTS asymptotic analysis
Homogenization method
Finite element algorithm
Error estimation

ABSTRACT

A new second-order two-scale (SOTS) analysis finite element algorithm is developed for the axisymmetric and spherical symmetric elastic problems with small periodic configurations. The axisymmetric structure considered is periodic in both radial and axial directions and homogeneous in circumferential direction, and the spherical symmetric structure is only periodic in radial direction and homogeneous in other two directions. The SOTS asymptotic expansions for the space problem, plane axisymmetric problem, and spherical symmetric problem are presented, and the main feature is that the anisotropic material is obtained by the homogenization. The analytical expressions of the cell functions and homogenized solutions for plane axisymmetric and spherical symmetric problems are obtained, and the error estimations of the expansions are established. The second-order asymptotic analysis finite-element algorithm is presented and the numerical examples are solved including the hollow cylinder, rotating disk and hollow sphere composed of periodic composite materials. The computational results demonstrate the effectiveness and accuracy of the SOTS asymptotic analysis algorithm, and the converging behavior of the asymptotic analysis algorithm agrees well with the theoretical prediction. It is also indicated that the stress distributions can be correctly computed only by adding the second-order correctors.

1. Introduction

With the rapid development of aeronautic and aerospace engineering, composite materials are widely used in the engineering because of the good thermal stability, high specific stiffness and strength. The composites are mostly heterogeneous with rapidly varying material coefficients. Success in practical application of composites largely depends on the possibility to predict their properties and behaviors through the development of appropriate models.

The evaluation of the thermal and mechanical behaviors for the composite materials involves many basic scientific principles, including multi-physical fields, multi-scale correlation models, and high-performance multi-scale algorithms. In the recent decades, the researches on the multi-scale homogenization methods have attracted the attention of many authors. Homogenization approaches allow the researches to reduce the governing equations with rapidly varying coefficients to the equations for media with effective properties. It is convenient as it buries micro structures into coefficients and then the computations can be performed on the macro scale. Moreover, by proper correctors, the solutions with oscillating behavior can be reproduced effectively. Mathematically, Bensoussan et al. (1978) adopted the idea of asymptotic expansion and homogenization method, and then various theoretical analysis and practical applications are studied by Lions (1981), Oleinik et al. (1992), Allaire (1992), Jikov et al. (1994), and Cioranescu and Donato (1999), including homogenization in perforated domains(Oleinik et al., 1992; Cioranescu and Saint Jean Paulin, 1997), large deformation problem(Takano et al., 2000), and conductive- ratiative coupled heat transfer problem(Allaire and Ganaoui, 2009). Based on this research, various multi-scale methods have been proposed(Hou and Wu, 1997; E and Engquist, 2003), but only the first-order asymptotic expansions are considered. Cui and Cao (1998) introduced the Second-Order Two-Scale (SOTS) analysis method to predict physical and mechanical behaviors of composite and perforated materials more accurately by considering the second-order correctors and corresponding finite element method is established (see, for instance, Chen and Cui, 2004; Cao et al., 2002; Cao and Cui, 2004; Cao, 2006). The SOTS method is extended to heat conduction (Su et al., 2010), linear elasticity (Su et al., 2011) problems with quasi-periodic structures, and the thermo-elastic problem is studied by Ma and Cui (2013). Wang and Cui (2014) developed the SOTS method for bending behavior analysis of

* Corresponding author at: Hypervelocity Aerodynamics Institute, China Aerodynamics Research and Development Center, Mianyang 621000, China.
Tel.: +86 10 82330957. Fax: +86-10-8231-7341.
E-mail address: zhli0097@x263.net, lizhihui@mail.tsinghua.edu.cn (Z. Li).

http://dx.doi.org/10.1016/j.ijsolstr.2015.08.030
0020-7683/

composite plate. The second-order asymptotic solutions for the integrated heat transfer problems with conduction, convection and radiation in periodic composite or porous materials are discussed by Zhang and Cui (2012), Yang et al. (2012), Ma and Cui (2013) and Yang et al. (2014). The multi-scale second-order solutions for the quasi-static and dynamic thermo-elastic problems are obtained by Feng and Cui (2004) and Wan (2007). By using the statistical SOTS method, the mechanical properties of materials with random distribution grains is studied by Li and Cui (2005) and the computation of the dynamic thermo-elastic behaviors for dynamic thermo-mechanical coupled response of random particulate composites is carried out by Yang et al. (2014).

For most literatures, the numerical simulations were often tested and carried out in the regular domain, and did not extend to more general and arbitrary domains. As we all known, the structures made of composites are not always regular, and the axisymmetric and spherical symmetric structures are often used in the engineering, such as the axisymmetric spacecraft, cylindrical pressure vessel, culvert pipe, tube of the artillery, or the high pressure sphere tank. Multilayered or periodic composite configurations are often utilized to increase the bearing capacity of the structure. To make the asymptotic analysis for this kind of structure, we should generalize the asymptotic analysis method to the elastic problems in cylindrical and spherical coordinate. Chatzigeorgiou et al. (2008) studied the homogenization for the hollow cylinder with discontinuous properties in the cylindrical coordinate system but the first or higher order correctors of the displacement were not discussed. In this paper, we focus on the elastic problems for the axisymmetric and spherical symmetric structures with periodic configurations and develop the second-order two-scale asymptotic expansions to simulate the fluctuating behaviors of the displacement and stress fields. For the purpose, this paper is outlined as follows. The elastic problems for the axisymmetric and spherical structures are presented and the equivalent compact formulations are given for the convenience of the second-order asymptotic analysis in Section 2. The second-order asymptotic expansions of the displacement are formally defined, the explicit expressions for the cell functions and homogenized solutions are obtained and the error estimations for the plane axisymmetric and spherical symmetric problems are discussed in Section 3. The finite element procedure is presented in Section 4. Several numerical examples are illustrated in Section 5, followed by the conclusions and expectation for the future work in Section 6. Throughout this paper, convention of summation on repeated indices is adopted and the common notations of Sobolev spaces are used for the analysis. The letters in bold represent the matrix or vector functions in the formulation. By $O(\varepsilon^k)$, $k \in \mathbb{N}$, we denote that there exists a constant c independent of ε and $|O(\varepsilon^k)| \le c\varepsilon^k$.

2. Governing equations

2.1. Axisymmetric elastic problem

In the three-dimensional axisymmetric domain Ω with symmetric plane A, set the cylinder coordinate as (x_1, θ, x_2) instead of the conventional (r, θ, z) for the convenience of the second-order asymptotic analysis. Consider the composite structures shown in Fig. 1. The hollow cylinder in Fig. 1(a) is multilayered with periodic piecewise constant elastic coefficients while there are periodic distributions in both radial and axial direction in the cylinder in Fig. 1(b), both of which are homogeneous in the θ direction.

The linear elastic problem in the axisymmetric plane can be formulated, as follows:

$$\begin{cases} -\dfrac{1}{x_1}\dfrac{\partial}{\partial x_1}(x_1\sigma_1^\varepsilon) - \dfrac{\partial}{\partial x_2}(\sigma_{12}^\varepsilon) + \dfrac{1}{x_1}\sigma_\theta^\varepsilon = f_1^\varepsilon(\boldsymbol{x}) \quad in \quad A, \\ -\dfrac{1}{x_1}\dfrac{\partial}{\partial x_1}(x_1\sigma_{12}^\varepsilon) - \dfrac{\partial}{\partial x_2}(\sigma_2^\varepsilon) = f_2^\varepsilon(\boldsymbol{x}) \quad in \quad A, \\ \boldsymbol{u}^\varepsilon = 0 \quad \text{on} \quad \Gamma_1, \quad \sigma_{ij}^\varepsilon n_j = p_i \quad \text{on} \quad \Gamma_2, \end{cases} \tag{1}$$

in which $\boldsymbol{u}^\varepsilon = [u_1^\varepsilon \;\; u_2^\varepsilon]^T$ denotes the displacement vector with u_1^ε and u_2^ε as the radial and axial components, respectively. The superscript "T" denotes the transpose operator and ε is a small parameter related to periodicity. $\boldsymbol{f}^\varepsilon = [f_1^\varepsilon \;\; f_2^\varepsilon]^T$ is the volume force. Define the stress vector $\boldsymbol{\sigma}^\varepsilon$ by

$$\boldsymbol{\sigma}^\varepsilon = [\sigma_1^\varepsilon \;\; \sigma_2^\varepsilon \;\; \sigma_\theta^\varepsilon \;\; \sigma_{12}^\varepsilon]^T,$$

and the strain vector $\boldsymbol{e}^\varepsilon$ by

$$\boldsymbol{e}^\varepsilon = [e_1^\varepsilon \;\; e_2^\varepsilon \;\; e_\theta^\varepsilon \;\; e_{12}^\varepsilon]^T = \left[\frac{\partial u_1^\varepsilon}{\partial x_1} \quad \frac{\partial u_2^\varepsilon}{\partial x_2} \quad \frac{u_1^\varepsilon}{x_1} \quad \frac{\partial u_1^\varepsilon}{\partial x_2} + \frac{\partial u_2^\varepsilon}{\partial x_1}\right]^T.$$

Also define the stress tensor σ_{ij}^ε and

$$\sigma_{11}^\varepsilon = \sigma_1^\varepsilon, \quad \sigma_{22}^\varepsilon = \sigma_2^\varepsilon, \quad \sigma_{21}^\varepsilon = \sigma_{12}^\varepsilon.$$

The constitutive relation can be expressed as

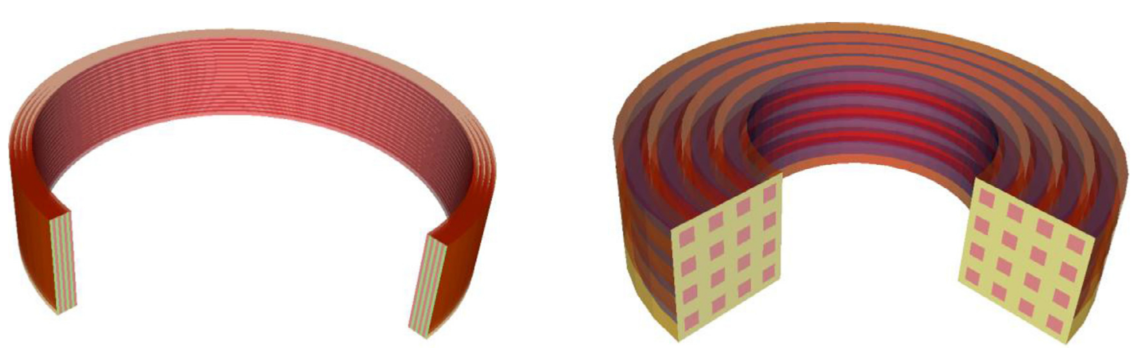
$$\boldsymbol{\sigma}^\varepsilon = \boldsymbol{D}^\varepsilon \boldsymbol{e}^\varepsilon,$$

with the elastic constitutive matrix $\boldsymbol{D}^\varepsilon$

$$\boldsymbol{D}^\varepsilon = \begin{bmatrix} \lambda^\varepsilon + 2\mu^\varepsilon & \lambda^\varepsilon & \lambda^\varepsilon & 0 \\ \lambda^\varepsilon & \lambda^\varepsilon + 2\mu^\varepsilon & \lambda^\varepsilon & 0 \\ \lambda^\varepsilon & \lambda^\varepsilon & \lambda^\varepsilon + 2\mu^\varepsilon & 0 \\ 0 & 0 & 0 & \mu^\varepsilon \end{bmatrix}. \tag{2}$$

λ^ε and μ^ε are the Lamé coefficients of the material expressed by Young's modulus E^ε and Possion's ratio ν^ε as

$$\lambda^\varepsilon = \frac{E^\varepsilon \nu^\varepsilon}{(1-2\nu^\varepsilon)(1+\nu^\varepsilon)}, \quad \mu^\varepsilon = \frac{E^\varepsilon}{2(1+\nu^\varepsilon)}. \tag{3}$$



(a) Multilayered hollow cylinder

(b) Axisymmetric structure with periodicity in two directions

Fig. 1. Axisymmetric composite structures.