International Journal of Solids and Structures 78 (2016) 160–173

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

On the nature of pressure dependence in foams

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ARTICLE INFO

Article history:
Received 16 October 2014
Revised 22 January 2015
Available online 22 October 2015

Keywords:
Foam
Yielding
Strain energy
Deformation mode
Pressure dependence

ABSTRACT

Present study explores the physical background of pressure dependence observed in the yield behavior of solid foams by analyzing the partition of strain energy and stress distribution in the struts that make up the foam. To this end, transversely isotropic Kelvin foam models of three different relative densities are utilized in FE analysis to probe the yield surface along various biaxial and triaxial stress paths for a wide range of mean stress. From a macroscopic viewpoint, it is observed that in addition to well-documented quadratic dependence of yielding on mean stress there also exists a linear pressure dependence, which has not been properly addressed in the literature. Partition of strain energy at the time of macroscopic yielding into bending and stretch modes of deformation within struts provides a unique tool with which the dominant deformation mode that drives yielding can be identified as a function of the stress path followed. Analysis of FE results indicates that positive mean stress, as compared to negative mean stress, provides a strong configurational stability in deformation kinematics that limits bending mode and promotes the stretch mode of deformation in struts. Increasing the fraction of strain energy stored in stretch mode effectively increases the critical strain energy of yielding and, thereby, delays the onset of microscopic yielding. Negative mean stress, on the other hand, results in a weak configurational stability where bending mode is more prominent. Furthermore, it seems that the degree of this already weak stability quickly decays with the magnitude of negative mean stress and completely disappears when (and if) the stabilizing effect of joint stiffness is exceeded. We conclude that this contrasting behavior is the main source of (i) linear pressure dependence observed in the yield behavior as well as (ii) the stronger effect of linear pressure in low-density foams.

1. Introduction

Growing use of cellular solids in diverse and increasingly demanding engineering applications, mainly because of their high specific strength and energy absorption capacity, requires a better understanding and modeling of their mechanical behavior particularly under multiaxial states of stress. Although mostly phenomenological in nature, substantial work on modeling the constitutive response of foams (PVC, PS, PP, Metallic) can be found in the literature (Deshpande and Fleck, 2000; Gioux et al., 2000; Miller, 2000; Gibson and Ashby, 2001; Gdoutos et al., 2002; Doyoyo and Wierzbicki, 2003; Xue and Hutchinson, 2004; Alkhader and Vural, 2009a; Ayyagari and Vural, 2015). Common to majority of these models is the recognition that yielding in foams has a quadratic dependence on the mean stress (pressure). Most recent works by authors Ayyagari and Vural (2015) and Shafiq et al. (2015) attempt to bring a clear physical justification to this long-observed quadratic pressure dependence by proposing that the total elastic strain energy density is 'the' entity governing the yield behavior in solid foams.

Furthermore, the majority of solid foams also exhibit tension/compression asymmetry in their yield behavior, which has been substantiated by numerous experimental investigations (Andrews et al., 1999; Bastawros et al., 2000; Deshpande and Fleck, 2000, 2001; Flores-Johnson et al., 2008; Gdoutos et al., 2001, 2002; Gioux et al., 2000; Gong et al., 2005a; Li et al., 2000; Ruan et al., 2007). However, in their attempt to capture these experimental observations, most of the proposed yield models rely on defining maximum stress caps based on theoretical buckling and/or brittle fracture loads. A noteworthy drawback to such an approach emerges when defining the flow and hardening rules to uniquely establish subsequent yield surfaces. Moreover, most recent biaxial and triaxial experimental data (Shafiq et al., 2015) for Divinycell H-100 PVC foam do not support the existence of sharp buckling caps as a means to justify tension-compression strength asymmetry in solid foams. Ayyagari and Vural (2015) tackle this difference in tensile and compressive yield behavior by introducing linear pressure dependence in their yield model, in addition to the quadratic dependence widely recognized in other models. Although the introduction of linear pressure dependence is rigorously justified by both experimental (Shafiq et al., 2015) and computational (Ayyagari and Vural, 2015) yield data in both biaxial and triaxial stress spaces, the nature of such linear pressure dependence still remains to be addressed.

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http://dx.doi.org/10.1016/j.ijsolstr.2015.09.002

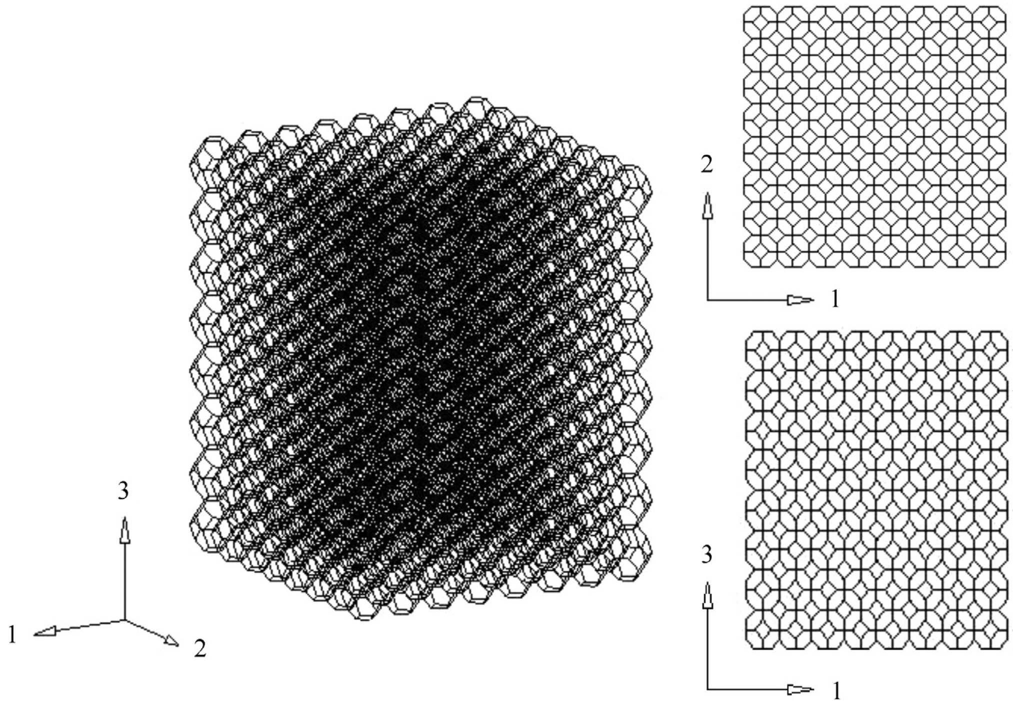

Fig. 1. Tetrakaidecahedral (Kelvin) foam model used in FE simulations. The model is geometrically stretched by a factor of 1.23 along 3-direction in order to introduce transverse isotropy such that 1–2 is the plane of isotropy while 1–3/2–3 are the planes of anisotropy.

Current work is an attempt to demonstrate the linear pressure dependence of yield behavior in solid foams under biaxial and triaxial stress fields as well as to elucidate underlying physical mechanisms of such a linear pressure dependence through FE analyses. To this end we use a periodic Kelvin foam model, which is geometrically stretched in one direction (see Fig. 1) to introduce transverse isotropy commonly observed in commercial foams. One must note that commercially available solid foams have stochastic structures where individual cells are reminiscent of tetrakaidecahedral (Kelvin) cells. Apart from this morphological similarity, the fact that average strut connectivity in stochastic foams is almost the same as the connectivity in periodic Kelvin structures brings out a topological similarity as well. Therefore, on modeling front, low connectivity periodic tetrakaidecahedral architecture has been the workhorse, for years, of both analytical and computational analyses of solid foams (similarly, periodic hexagonal architecture has been extensively used for 2D analyses). It is particularly this topological feature, i.e. low connectivity, that justifies the extensive use of periodic Kelvin cells in understanding and modeling the mechanical behavior of solid foams because it causes a bending dominated deformation mode at strut level (Desphande et al., 2001; Alkhader and Vural, 2009b). Periodic tetrakaidecahedral foam model has been the subject of many studies (e.g., Zhu et al., 1997; Warren and Kraynik, 1997; Gong et al., 2005a, 2005b; Luxner et al. 2007; Jang et al., 2010), and is considered as the first step towards understanding the behavior of real foams.

It must be noted at this point that periodic Kelvin models based on unit/characteristic cell as well as finite domains must be used with caution for the analysis of post-yield behavior at and beyond the initial stress peak (commonly observed for some compression dominated loading paths) where the onset of elastic instability and subsequent localized deformation may govern the mechanical response particularly for low density foam models. Laroussi et al. (2002) and Gong et al. (2005b) presented an analysis for this type of instability-driven complex response by using Bloch wave method and established the failure surface for some compressive triaxial loading paths. With this method, an array of symmetric as well as non-symmetric buckling modes, which are also affected by cell anisotropy and loading path, can be identified at the onset of instability and subsequent localization. Present study neglects the analysis of such elastic bifurcations for two reasons: (1) the macroscopic yield point of foam model is determined by a standard offset strain technique, and the resulting yield point always occurs before the initial stress peak – if there is any – and onset of any potential instability; (2) normalized yield stresses under uniaxial compression are much lower in current study (by a factor of 5–7) than the normalized elastic bifurcation stresses reported in Laroussi et al. (2002) for various relative densities in 0.02–0.10 range and also in Gong et al. (2005b) for the relative density of 0.025. Therefore, the yield behavior of the Kelvin foam model in current study is akin to those reported in Jang and Kyriakides (2009b) and Jang et al. (2010) where the limit load is dictated by plastic ligament bending and gradual reduction in stiffness at the periodic cell level rather than elastic instability.

Although, the categorization of cellular structures based on their deformation modes such as *bending dominated* or *stretch dominated* has been studied by Desphande et al. (2001), the influence of stress state on the distribution of deformation modes within a particular structure (or in structures with varying topologies) has not been quantitatively studied. Unlike our most recent work (Ayyagari and Vural, 2015; Shafiq et al., 2015), where establishing and modeling of the macroscopic yield surface were the main subjects of investigation, the current study investigates the partition of elastic strain energy within the struts of foam structure into bending and stretch modes with the aim of bringing a physical and mechanistic insight into the linear pressure dependence observed in the yield behavior of solid foams (including, but not limited to, uniaxial tension/compression asymmetry). To this end, a transversely isotropic Kelvin foam model is constructed and used in FE analyses to explore the distinctions in mechanical response at ligament scale as a function of stress multiaxiality.

2. FE modeling and boundary conditions

Finite element (FE) analyses were performed using ABAQUS implicit commercial software. Periodic tetrakaidecahedral model with