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On the dynamic behaviour of a two-dimensional elastic metamaterial system

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ABSTRACT

A two-dimensional mechanical system with local resonators is proposed as an elastic metamaterial model that exhibits both negative mass and negative modulus under specific frequencies. The proposed representative cell, which consists of a series of properly arranged local elements, can generate controllable translational and rotational resonance. By adjusting the local property of the element negative mass and negative modulus can be achieved independently. The dynamic behaviour of the developed metamaterial system under different loading frequencies is studied in detail and the effective mass and effective elastic modulus under different loading frequencies are evaluated. Examples of harmonic elastic waves in this two-dimensional metamaterial system are studied to illustrate the effect of the generated negative mass and/or modulus on wave propagation.

1. Introduction

Metamaterials are engineered materials exhibiting unique properties, which are not commonly seen or physically inconceivable in nature. Early efforts in developing such unusual materials had been mostly focussed on new electromagnetic materials to control electromagnetic waves. It was demonstrated theoretically that materials with simultaneous negative permittivity and negative permeability under certain frequencies will possess a negative refractive index (Veselago, 1968), from which the concept of metamaterial was proposed. Due to the limitation of technologies in synthesis and fabrication, this novel idea remained to be an academic curiosity at that time. Only in recent years the feasibility of designing this type of metamaterials, so-called left-handed electromagnetic materials, was justified and the new materials were successfully devised to achieve effectively negative permittivity and negative permeability (Smith et al., 2000; Pendry, 2000; Shelby et al., 2001). By making use of the unusual properties of the developed materials, researchers are considering using metamaterials in advanced applications, which could not be achieved before, such as invisible cloak (Pendry et al., 2006; Schuring et al., 2006) and hyperlens (Liu et al., 2007) for electromagnetic waves.

As the counterpart of electromagnetic materials in mechanical engineering, in recent years, acoustic or elastic metamaterials, which exhibit negative effective mass or negative modulus, have also received significant attention. Many peculiar properties of these metamaterials have been predicted based on different material models, which provide the mathematical analogy between electromagnetic behaviour and mechanical behaviour. The main focus in the development of these materials is the design of engineered microstructures, which can generate the desired properties. The first design of such a material structure is by embedding heavy spheres coated with soft silicon rubber in epoxy to acquire, experimentally, negative mass at certain loading frequencies (Liu et al., 2000). The underlying mechanism of this material structure is the local mechanical resonance. This mechanism has been used in the design of other metamaterial systems, such as distributing local mechanical resonators in water (Larabi et al., 2007) to develop acoustic metamaterials, or using distributed membranes to generate periodic resonators in acoustic media to achieve negative mass (Cselyuszka et al., 2015; Lee et al., 2009a; Mei et al., 2012). It is now well-established that properly designed local resonance can generate negative effective mass for acoustic metamaterials. In general the effective mass will be frequency-dependent and become negative at certain frequencies (Huang et al., 2009; Liu et al., 2005; Milton and Willis, 2007; Yao et al., 2008; Movchan and Slepyan, 2007). Recently, multilevel resonators have also been considered in the generation of negative mass in a one-dimensional hierarchical metamaterial system (An et al., 2015).

Acoustic metamaterials with apparent negative modulus have also received increasing interests. A one-dimensional acoustic metamaterial system consisting of a chain of subwavelength Helmholtz resonators experimentally yields negative group velocity as a result of the negative bulk modulus (Fang et al., 2006). Similar results have also been achieved in a metamaterial system consisting

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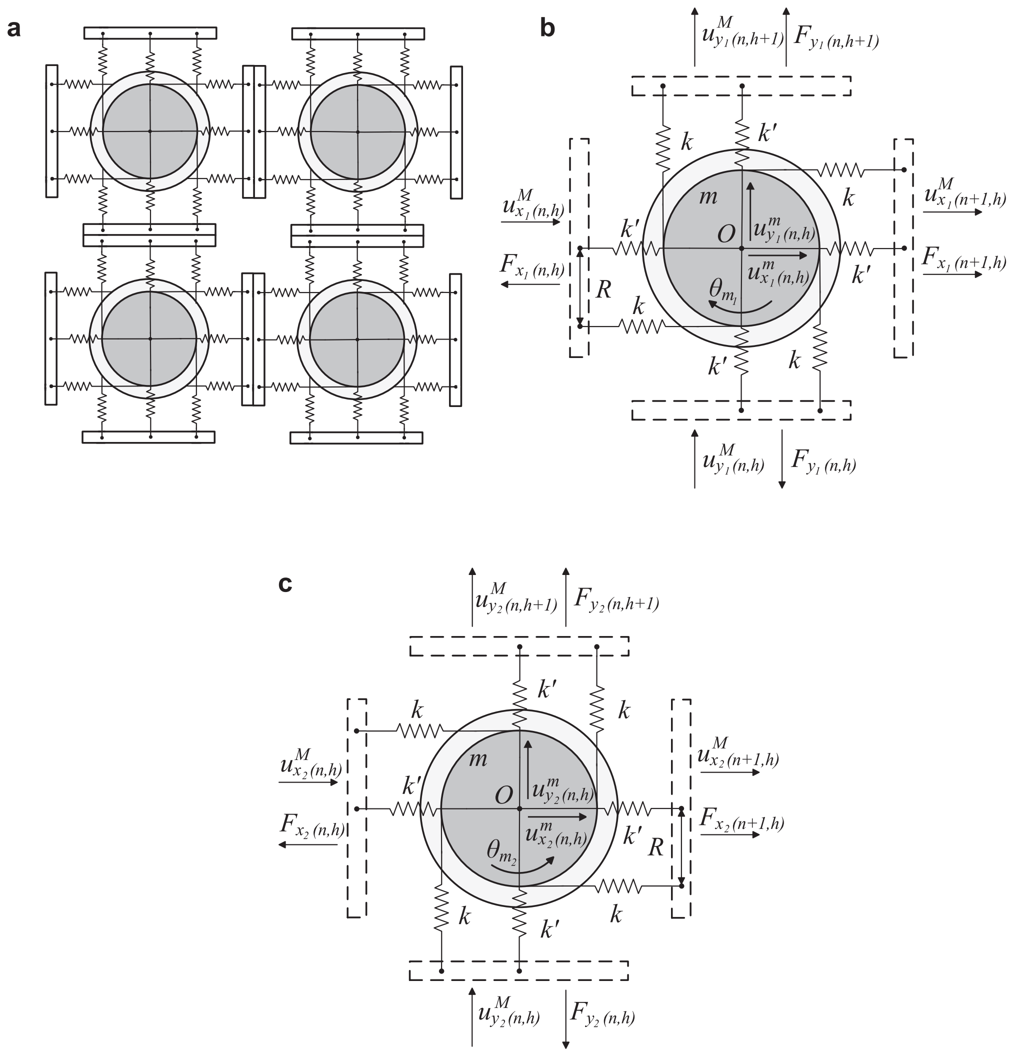

Fig. 1. The two-dimensional metamaterial system: (a) periodic cells, (b) cell unit 1, (c) cell unit 2.

of an array of tubes with side holes (Lee et al., 2009b). To develop elastic metamaterials, theoretical models have been developed based on spring-mass systems, mainly in one dimension cases, exhibiting frequency-dependent effective modulus (Huang and Sun, 2011; Zhou et al., 2012). Attempts have also been made to achieve both negative mass and negative modulus (Liu et al., 2011; Bigoni et al., 2013) using more general two-dimensional spring-mass models. However, in these works, negative mass and negative modulus cannot be adjusted independently in a controllable manner. It should be mentioned that one of the main issue in the development of this type of metamaterials is to precisely manipulate the dynamic behaviour and wave propagation in the metamaterials, and therefore, the desired negative mass and negative modulus should be reliably controlled. Recently, a one-dimensional metamaterial model formed with a special mass-spring system is proposed, with which the effective mass and effective modulus, depending on the loading frequency and structural parameters, can be easily tuned to desired positive or negative values by adjusting the geometry and material properties of the system (Wang, 2014).

The current paper, as the extension of the corresponding study of the one-dimensional model (Wang, 2014), presents a two-dimensional(2D) metamaterial system consisting of periodic cells formed by properly arranged mass-spring structures. The structure allows rather independent local translation motion and rotation motion, making effective mass and effective modulus being easily controlled, with the translational motion directly dominating the effective mass and the rotational motion controlling the effective modulus. Numerical simulation is conducted to show the resulting negative mass and negative modulus under specific loading frequencies and geometries. The frequency response of the effective properties and the effects of geometric parameters are also presented to illustrate the feasibility of designing the desired effective properties by proper adjusting the metamaterial cell. The dynamic behaviour of such a two-dimensional matamaterial system is also studied by considering the elastic wave propagation in the effective medium under different conditions of the metamaterial cell. Some of the basic features of the current 2D model are similar to that of the previous 1D model, but significantly different dynamic behaviour in 2D cases is observed, such as the Poisson's ratio effect, the direction dependence of wave propagation, and the complexity of the 2D model. More importantly, the 2D model will provide a guideline for designing realistic 2D meta materials.

2. Formulation of a two-dimensional metamaterial model

Consider a two-dimensional metamaterial system formed by a periodic structure, as shown in Fig. 1(a). The representative cell of the system consists of two separate units, as shown schematically in Fig. 1(b and c). Each unit contains one circular rigid body, four linear springs attached to the centre, and four side springs wrapping around the rigid body with a radius R. These springs are attached to