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Transient dynamic stress intensity factors around three stacked parallel cracks in an infinite medium during passage of an impact normal stress

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A B S T R A C T

Transient dynamic stresses around three stacked parallel cracks in an infinite elastic plate are estimated for an incident impact stress wave impinging normal to the cracks. Using Fourier and Laplace transform techniques, the boundary conditions are reduced to six simultaneous integral equations in the Laplace domain. The differences in the displacements inside the cracks are expanded in a series of functions that have zero value outside the cracks. The Schmidt method is used to solve the unknown coefficients in the series such that the conditions inside the cracks are satisfied. The stress intensity factors are defined in the Laplace domain, and these are inverted using the numerical method. The stress intensity factors are calculated numerically for some crack configurations.

1. Introduction

Mechanical members are inevitably weakened by cracks. If a load is suddenly applied to the members, then an estimate of the transient stress intensity factors becomes important. Ravera and Sih (1970) estimated the Mode III dynamic stress intensity factor for a finite crack in an infinite elastic medium under an impact incident stress wave. Later, Sih et al. (1972) solved the corresponding Mode I and Mode II stress intensity factors for a crack in an infinite elastic medium. Thau and Lu (1971) used the Wiener–Hopf method to solve the dynamic transient problem for a crack in an infinite medium. Their solutions are exact from the moment an incident stress wave arrives at a crack end, until a diffracted P wave reaches the opposite crack end, is rediffracted, and then returns to the original edge.

Materials are occasionally weakened by parallel cracks. In this case, it is important to clarify the effect of these cracks on the transient dynamic stress intensity factors. Takakuda et al. (1984) estimated the transient dynamic stress intensity factors for two stacked parallel cracks in an infinite plane subjected to an impact anti-plane shear stress wave. Itou (1995) solved the transient dynamic problem for two parallel cracks in an infinite isotropic plate during the passage of impact shock stress waves. Furthermore, Itou (2015) also estimated the transient dynamic stress intensity factors for three parallel cracks, in which two collinear cracks were situated above the lower center crack. Wu et al. (2015) obtained the Mode III dynamic stress intensity factors for three stacked parallel cracks in an infinite medium. The materials are likely loaded by an impact normal stress rather than anti-plane dynamic loading. Therefore, it is appropriate to estimate the Mode I and II stress intensity factors for three stacked parallel cracks under a normal incident impact stress.

In the present paper, the transient dynamic stresses are solved for three stacked parallel cracks under an incident normal impact stress. Using the Fourier transform, the mixed boundary value conditions concerning the stress field are reduced to a set of dual integral equations in the Laplace domain. To solve these integral equations, the differences in displacement at the upper, middle, and lower cracks are expanded in a series of functions that have zero value outside the cracks. Six sets of infinite series result from this process, each of which contains an infinite number of unknown coefficients. Solving the unknown coefficients is a difficult task. In the previous work (Itou, 2010), the corresponding time-harmonic stresses were solved for three stacked parallel cracks in an infinite elastic plate during the passage of time-harmonic stress waves propagating normal to the cracks. Using the Schmidt method developed in Itou (2010), in the present work the unknown coefficients in the six sets of infinite series are determined in the Laplace domain so that they satisfy the conditions inside the three cracks.

As the stress intensity factors are defined in the Laplace domain, they are inverted to the physical domain using Miller and Guy's numerical technique (1996). The stress intensity factors are calculated numerically for several crack configurations.

2. Fundamental equations

Consider a crack in an infinite plate located along the x−axis from $-a$ to a at $y = 0$, with respect to the rectangular coordinates (x, y).

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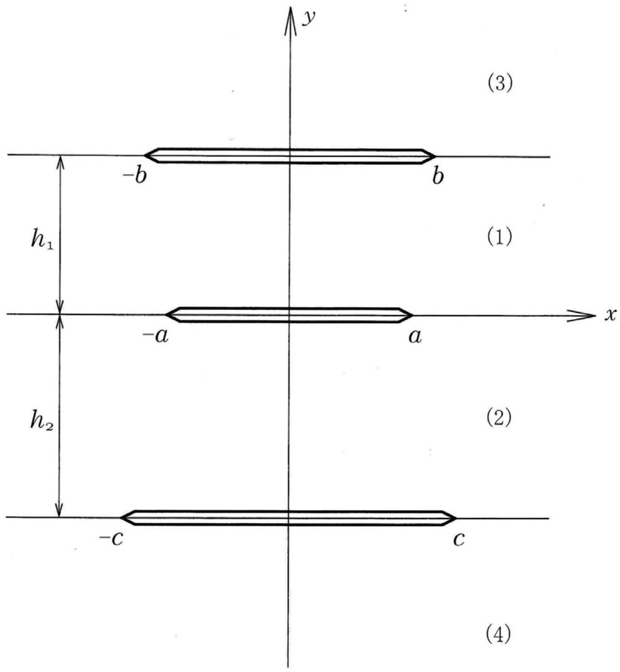

Fig. 1. Geometry and coordinate system.

There is also an upper crack from $-b$ to b at $y=h_1$, and a lower crack from $-c$ to c at $y=-h_2$. These cracks are shown in Fig. 1. For convenience, $0<y<h_1$ is referred to as layer (1), $-h_2<y<0$ is referred to as layer (2), $h_1<y$ is referred to as the upper half-plane (3), and $y<-h_2$ is referred to as the lower half-plane (4).

Let u and v be defined as the x and y components of the displacement, respectively. If the displacement components u and v are expressed by two functions $\varphi(x,y,t)$ and $\phi(x,y,t)$ such that

$$u=\partial\varphi/\partial x-\partial\phi/\partial y,\quad v=\partial\phi/\partial x+\partial\varphi/\partial y, \tag{1}$$

then the equations of motion reduce to the following forms:

$$\begin{aligned}&\partial^2\varphi/\partial x^2+\partial^2\varphi/\partial y^2=1/c_L^2\times\partial^2\varphi/\partial t^2,\\&\partial^2\phi/\partial x^2+\partial^2\phi/\partial y^2=1/c_T^2\times\partial^2\phi/\partial t^2\end{aligned} \tag{2}$$

where t is time. The dilatational wave velocity c_L and the shear wave velocity c_T under plane stress conditions can be given as follows:

$$c_L^2=2\mu/[(1-\nu)\rho],\quad c_T^2=\mu/\rho. \tag{3}$$

where μ is the modulus of rigidity, $\nu$is Poisson's ratio, and ρ is the density of the material.

The stresses can be expressed by the equations

$$\begin{aligned}\tau_{yy}/(2\mu)&=-\partial^2\varphi/\partial x^2+\kappa^2/(2c_L^2)\partial^2\varphi/\partial t^2+\partial^2\phi/\partial x\partial y\\\tau_{xx}/(2\mu)&=-\partial^2\varphi/\partial y^2+\kappa^2/(2c_L^2)\partial^2\varphi/\partial t^2-\partial^2\phi/\partial x\partial y\\\tau_{xy}/(2\mu)&=\partial^2\varphi/\partial x\partial y+\partial^2\phi/\partial x^2-\kappa^2/(2c_L^2)\partial^2\varphi/\partial t^2\end{aligned} \tag{4}$$

with

$$\kappa^2=(c_L/c_T)^2=2/(1-\nu). \tag{5}$$

The incident stress waves that propagate through the infinite plate parallel to the y-axis in the negative direction are expressed as follows:

$$\begin{aligned}\tau_{yy}^{(inc)}&=pH[t+(y-h_1)/c_L]\\\tau_{xy}^{(inc)}&=0\end{aligned} \tag{6}$$

where p is a constant and $H(t)$ is the Heaviside unit function. We set $t=0$ when the wave front reaches the upper crack at $y=h_1$.

The boundary conditions for this problem can be expressed as

$$\tau_{yy1}=\tau_{yy3},\quad\tau_{xy1}=\tau_{xy3}\quad\text{at } y=h_1,\quad|x|\le\infty \tag{7}$$

$$\tau_{yy1}=\tau_{yy2},\quad\tau_{xy1}=\tau_{xy2}\quad\text{at } y=0,\quad|x|\le\infty \tag{8}$$

$$\tau_{yy2}=\tau_{yy4},\quad\tau_{xy2}=\tau_{xy4}\quad\text{at } y=-h_2,\quad|x|\le\infty \tag{9}$$

$$\tau_{yy1}=-pH[t],\quad\tau_{xy1}=0\quad\text{at } y=h_1,\quad|x|\le b \tag{10.1}$$

$$u_3-u_1=0,\quad v_3-v_1=0,\quad\text{at } y=h_1,\quad b\le|x| \tag{10.2}$$

$$\tau_{yy1}=-pH[t-h_1/c_L],\quad\tau_{xy1}=0\quad\text{at } y=0,\quad|x|\le a \tag{11.1}$$

$$u_1-u_2=0,\quad v_1-v_2=0,\quad\text{at } y=0,\quad a\le|x| \tag{11.2}$$

$$\begin{aligned}\tau_{yy2}&=-p\,[t-(h_1+h_2)/c_L],\\\tau_{xy2}&=0\quad\text{at } y=-h_2,\quad|x|\le c\end{aligned} \tag{12.1}$$

$$u_2-u_4=0,\quad v_2-v_4=0,\quad\text{at } y=-h_2,\quad c\le|x| \tag{12.2}$$

where the subscript $i\ (=1,\ 2)$ indicates the layer i, the subscript 3 indicates the upper half-plane (3), and the subscript 4 indicates the lower half-plane (4).

3. Analysis

To obtain a solution, the following Laplace transforms are introduced:

$$\begin{aligned}g^*(s)&=\int_0^\infty g(t)\exp(-st)\,dt,\\g(t)&=1/(2\pi i)\times\int_{Br.}g^*(s)\exp(st)\,ds\end{aligned} \tag{13}$$

as well as the following Fourier transforms:

$$\begin{aligned}\bar f(\xi)&=\int_{-\infty}^\infty f(x)\exp(i\xi x)\,dx,\\f(x)&=1/(2\pi i)\times\int_{-\infty}^\infty\bar f(\xi)\exp(-i\xi x)\,d\xi.\end{aligned} \tag{14}$$

Applying Eqs. (13) and (14) to Eq. (2), we obtain:

$$d^2\bar\varphi^*/dy^2+\gamma_1^2\bar\varphi^*=0,\quad d^2\bar\phi^*/dy^2+\gamma_2^2\bar\phi^*=0 \tag{15}$$

with

$$\gamma_1=\sqrt{\xi^2+(s/c_L)^2},\quad\gamma_2=\sqrt{\xi^2+(\kappa s/c_L)^2}. \tag{16}$$

For the layer $i\ (=1,\ 2)$, the solutions of Eq. (15) have the following forms:

$$\begin{aligned}\bar\varphi_i^*&=A_i\sinh(\gamma_1 y)+B_i\cosh(\gamma_1 y)\\\bar\phi_i^*&=C_i\sinh(\gamma_2 y)+D_i\cosh(\gamma_2 y)\end{aligned} \tag{17}$$

where A_i, B_i, C_i, and D_i are unknown coefficients. For the upper half-plane (3) and the lower half-plane (4), the solutions of Eq. (15) have the following forms in terms of the unknown coefficients C_3, D_3, C_4, and D_4:

$$\begin{aligned}\bar\varphi_3^*&=C_3\exp(-\gamma_1 y)\\\bar\phi_3^*&=D_3\exp(-\gamma_2 y)\end{aligned} \tag{18}$$

$$\begin{aligned}\bar\varphi_4^*&=C_4\exp(\gamma_1 y)\\\bar\phi_4^*&=D_4\exp(\gamma_2 y).\end{aligned} \tag{19}$$

The stresses and displacements can be expressed by twelve unknowns: $A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2, C_3, D_3, C_4$, and D_4. Using