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# Two-dimensional fretting contact analysis of piezoelectric materials

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ABSTRACT

This paper makes the first attempt to present a theoretical study on the fretting contact of a piezoelectric half-plane under a rigid cylindrical punch. It is assumed that the rigid punch is a perfect insulating body with zero electric charge distribution. The two bodies are brought into contact first by a monotonically increasing normal load, and then by a cyclic tangential load which is less than that necessary to cause complete sliding. The whole contact region is composed of an inner stick region and two outer slip regions in which Coulomb's friction law is assumed. Since the fretting contact problem is frictional and history dependent, therefore we first solve the normal loading phase, and then solve the tangential loading phase. With the use of Fourier integral transform technique and the superposition theorem, the problem is reduced to a set of coupled Cauchy singular integral equations. An iterative method is used to determine the unknown stick/slip region and contact tractions. The effects of the friction coefficient and radius of the punch on the normal contact pressure, tangential traction, in-plane stress and in-plane electric displacement are discussed during different loading phases in detail. The results indicate that the piezoelectric effect leads to the concentration of the normal contact pressure and tangential traction, which may cause a serious influence on the fretting contact damage.



## 1. Introduction

Two nominally-clamped surfaces subjected to vibration loads exhibit small-scale oscillatory tangential motion known as fretting. The surface damage so induced sometimes initiates and grows cracks in a phenomenon known as fretting fatigue. Contact damage and fretting fatigue, which occur at engineering structures, aircrafts, electric power, traffic tools, gas turbine engines, etc, are the main cause of the failure in the key components. For homogeneous elastic materials, the fretting contact problem has been extensively considered by many investigators. This problem was first solved by Cattaneo (1938) for a monotonically increasing tangential load and by Mindlin (1949) for a cyclically varying tangential load. In their analysis, the contact bodies were made of similar materials which were also called as the fretting contact of similar bodies. This kind of problem is uncoupled between the normal and tangential tractions, and therefore is easy to solve. Subsequently, Ciavarella (1998a, 1998b) and Ciavarella and Hills (1999) extended the problem to a general plane fretting contact between elastically similar bodies. For the fretting contact of dissimilar bodies, it is more complex than that of similar bodies because of the coupling in the contact. By using the self-similarity assumption, Spence (1973) considered the two-dimensional contact of an elastic homogeneous half-space by a rigid power law punch under a slowly applied normal load only. Keer and Farris (1987) discussed the influence of the finite thickness of the elastic layer and cyclic tangential loading on the contact stresses. Hanson et al. (1989) modeled the actual experimental situation of fretting fatigue and obtained the contact stress distributions, size and location of the stick region, relative displacements in the slip regions and energy dissipation. Nowell et al. (1988) and Hanson and Keer (1989) investigated the fretting contact between two dissimilar elastic cylinders under the normal loading and cyclic tangential loading. In special, both Goodman approximation solution and fully coupled solution were considered by Nowell et al. (1988). The more detailed summaries of the fretting contact of homogeneous elastic materials could be found in the review article (Hills and Sosa, 1999) and the research monographs (Johnson, 1985; Hills et al., 1993; Hills and Nowell, 1994).

Recently, Ke and Wang (2007a, 2007b) extended the fretting contact problem to functionally graded materials (FGMs). They considered the two-dimensional fretting contact between an FGM coated half-plane and a rigid cylindrical punch subjected to the normal and tangential loads. Their results indicated that FGM coatings would have potential applications in improving the resistance to the fretting contact damage at the contact surfaces. Later, Ke and Wang (2010) analyzed the problem involving normal and

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tangential loadings of two dissimilar elastic bodies with FGM coatings. Furthermore, the partial slip contact of FGMs under a monotonically applied normal load was considered by Kallel-Kamoun et al. (2010), Elloumi et al. (2010), and Chen and Chen (2013) for the two-dimensional case and Liu et al. (2012) for the axisymmetric case.

Piezoelectric materials, which possess intrinsic electro-mechanical coupling effects, are extensively used in many hi-tech areas such as electronics, laser, supersonics, microwave, navigation, biology, etc. In these applications, piezoelectric devices often service in the vibration environment, and are subjected to the concentrated contact loads. Therefore, fretting contact damage inevitably occurs in smart devices. For the sake of the safety of smart devices, we must develop the theoretical model for the fretting contact, and understand the fretting fatigue mechanism of piezoelectric materials. To the best of the authors' knowledge, so far, no work was reported on the fretting contact of piezoelectric materials despite of the importance for the application in smart devices. That is because the fretting contact problem is more complex than the frictionless or sliding frictional contact problem. Especially, the problem is fully coupled and stick–slip phenomenon occurs for two dissimilar bodies even under the normal loading alone. However, the frictionless contact and sliding frictional contact of piezoelectric materials have received considerable concerns and been solved perfectly.

Giannakopoulos and Suresh (1999) presented the general analytical theory for the frictionless contact of a piezoelectric half-space under both insulating and conducting punches. Using the complex potential function method, Chen (2000) considered the transversely piezoelectric half-space subjected to a rigid punch with an arbitrary profile. Wang et al. (2008) investigated the frictionless contact of a piezoelectric ceramic layer of finite thickness by a conducting or insulating punch. They also solved the stress and electric displacement intensity factors at the punch edges. Using the finite element method, Barboteu and Sofonea (2009) analyzed the frictionless contact between a piezoelectric body and an electrically conductive foundation. Zhou and Lee (2011) studied thermo-electro-mechanical contact behavior of a finite piezoelectric layer under a sliding punch with the frictional heat generation. They discussed effects of the material constants, relative sliding speed, frictional coefficient and finite thickness on the stress and temperature distributions. Recently, Ma et al. (2014) examined the two-dimensional sliding frictional contact of a piezoelectric half-plane under the action of a rigid flat or triangular punch. The contact problem of piezoelectric materials was also considered by Ramirez and Heyliger (2003), Wang et al. (2011), Zhou and Lee (2012) and Wu et al. (2012) for the frictionless case, Makagon et al. (2009) and Zhou and Lee (2014) for the sliding frictional case, and Chen and Yu (2005) and Guo and Jin (2009) for the adhesive case.

In this paper, we present a theoretic study on the two-dimensional fretting contact between a homogeneous transversely isotropic piezoelectric half-plane and a rigid insulting cylindrical punch. The two dissimilar bodies are first acted by a monotonically increasing normal load, and then by a cyclic tangential load. Since the fretting contact problem is frictional and history dependent, therefore the problem is handled by dividing the loading process into the normal loading phase and the tangential loading phase, respectively. The whole contact region is composed of an inner stick region and two outer slip regions in which Coulomb's friction law is assumed. With the aid of Fourier integral transform, the fretting contact problem is reduced to a set of coupled Cauchy singular integral equations which are then solved by using an iterative method to determine the unknown stick/slip regions and contact tractions. The effects of the friction coefficient and radius of the punch on the normal contact pressure, tangential traction, in-plane stress and in-plane electric displacement are discussed.

## 2. Fundamental solutions of the piezoelectric half-plane

Consider the problem shown in Fig. 1, where a homogeneous piezoelectric half-plane is acted by a normal concentrated line load $P$ and a tangential concentrated line load $Q$. The half-plane is the transversely isotropic homogeneous piezoelectric materials with poling in $z$-direction. The $z$-axis and $x$-axis are perpendicular and tangential to the surface, respectively. The linear piezoelectric constitutive equations under the plane strain state can be expressed in terms of displacement components, $u_x$ and $u_z$, and electric potential, $\phi$, as

$$\sigma_{xx} = c_{11}\frac{\partial u_x}{\partial x} + c_{13}\frac{\partial u_z}{\partial z} + e_{31}\frac{\partial \phi}{\partial z}, \tag{1}$$

$$\sigma_{zz} = c_{13}\frac{\partial u_x}{\partial x} + c_{33}\frac{\partial u_z}{\partial z} + e_{33}\frac{\partial \phi}{\partial z}, \tag{2}$$

$$\sigma_{xz} = c_{44}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) + e_{15}\frac{\partial \phi}{\partial x}, \tag{3}$$

$$D_x = e_{15}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) - \varepsilon_{11}\frac{\partial \phi}{\partial x}, \tag{4}$$

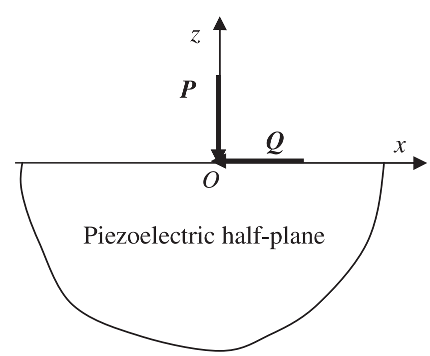


**Fig. 1.** A piezoelectric half-plane subjected to a normal concentrated line load $P$ and a tangential concentrated line load $Q$.

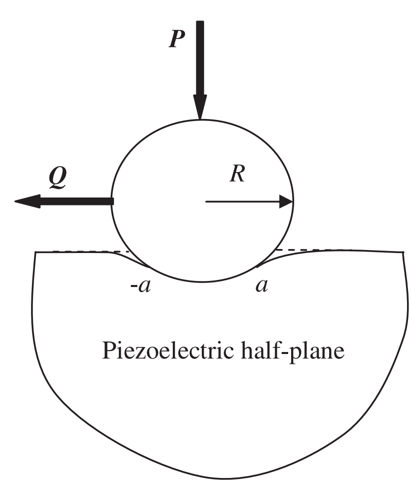


**Fig. 2.** Sketch map of the frictional contact between a homogeneous piezoelectric half-plane and a rigid cylindrical punch.