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Transition radiation excited by a surface load that moves over the interface of two elastic layers

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A B S T R A C T

Transition radiation is emitted when a perturbation source such as an electric charge or mechanical load, which has no inherent frequency, moves along a straight line at constant velocity in or near an inhomogeneous medium. Transition radiation of elastic waves is emitted, for example, by the wheels of a train due to track inhomogeneities such as non-uniform subsoil. This type of radiation was analyzed in the framework of several 1-D and 2-D elastic systems, but very few studies focus on elastic continua. Here, we consider a continuum consisting of two elastic layers (i.e., waveguides) that are coupled at a vertical interface. Both layers are in plane strain, have a free surface and are fixed to a rigid bottom, while the load is assumed to move along the free surface and over the layer interface. The response in each layer consists of the stationary eigenfield that moves with the load and a free field that propagates independently. The latter is expanded into a set of propagating and evanescent guided modes, while all fields are coupled at the interface. Orthogonality relations are employed to find the modal coefficients. Results show that the transition radiation energy (i.e., the energy flux through a surface far away from the layer interface) becomes powerful for load velocities approaching the critical velocity or for high contrast in material parameters. Furthermore, the free-field contribution to the energy flux through a circular surface close to the layer interface exhibits peculiar directivities. Depending on the contrast in material parameters and the load velocity, it can be extreme along the free surface, along the layer interface or into the medium. The free-field contribution can dominate that of the eigenfield for high load velocities, as is the case for the transition radiation energy. To study the features of the generated wave field in pure form, we take the load velocity sub-critical throughout the paper, excluding other radiation effects. However, the adopted solution is not restricted to that and is expected to work also for layered media, even when connected to other elastic structures like beams.

1. Introduction

Transition radiation is emitted when a perturbation source, which does not possess an inherent frequency, moves along a straight line at a constant velocity in an inhomogeneous medium or near such a medium (Ginzburg and Tsytovich, 1990). This phenomenon was described for the first time by Ginzburg and Frank (1946), who analyzed radiation of electromagnetic waves by a charged particle crossing the boundary between an ideal conductor and a vacuum. In these early studies concerned with transition radiation, it has already been explained that this phenomenon is universal from the physical point of view, meaning that it occurs irrespective of the physical nature of the source (i.e., it can be an electric charge, an acoustic monopole, a mechanical load, etc.).

The first study on transition radiation of elastic waves was published by Vesnitskii and Metrikin (1992). Such radiation is emitted, for example, by a train running on a conventional railway track. The wheels of the train, pressed against the rails by gravity, excite elastic waves in the railway due to track inhomogeneities such as sleepers, non-uniform subsoil and bridge supports. A review of the early studies on transition radiation of elastic waves in 1-D and 2-D elastic systems (i.e., strings, beams, membranes and plates) can be found in Vesnitskii and Metrikin (1996). Recently, the problem of a beam resting on an inhomogeneous Winkler foundation again attracted attention due to the introduction of some new solution methods; i.e., different types of modal expansion techniques (Dimitrovová and Varandas, 2009; Dimitrovová, 2010) and the moving-element method (Ang and Dai, 2013). Furthermore, non-linear springs were incorporated aiming at a more realistic description of the railway track substructure (Varandas et al., 2011, 2014). Transition radiation was also studied using 3-D

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finite-element or combined boundary-element and finite-element models for evaluating the effect of specific measures to mitigate amplified responses; e.g., Galvín et al. (2010) and Shan et al. (2013). Finally, transition radiation was explicitly addressed – apart from possibly giving rise to vehicle instability and passenger discomfort – as one of the causes of track and foundation degradation due to the associated and often strong amplification of the stress and strain fields (Coelho et al., 2011; Steenbergen, 2013; Varandas et al., 2014).

Transition radiation of waves in an elastic continuum was first theoretically described by van Dalen and Metrikine (2008). Two elastic half-planes were considered under the action of a constant, uniformly moving load that crosses the interface between the half-planes along the path normal to this interface. Though the chosen model has no direct practical application, the study provided physical insight into the mechanism of transition radiation in an elastic continuum. Compressional and shear waves as well as interface waves (i.e., Stoneley waves) can be excited, while the radiation spectra of the former show peculiar directivities, which is due to the coupling of the radiated waves at the interface.

In the current paper, we theoretically describe the phenomenon of transition radiation in a more realistic continuum model consisting of two elastic layers (i.e., waveguides) that are coupled at a vertical interface. Both layers have a free surface and are fixed to a rigid bottom, while the load is assumed to move along the free surface and over the layer interface, as shown in Fig. 1. There are two distinct differences compared to the above-discussed continuum model (van Dalen and Metrikine, 2008). First, the presence of the free surface allows the existence of surface waves. Second, the finite depth of the model requires a different solution method. The generated wave field in each layer is expanded into a complete set of propagating and evanescent guided modes (Stange and Friederich, 1992; Besserer and Malischewsky, 2004). The wave fields in both layers are coupled at the interface, which thus constitutes an interaction or coupled problem where each mode in one layer is coupled to all modes of the other. The semi-analytical solution is finally used for the aim of studying the main features of the wave field associated with the transition process.

In particular, after deriving the solution (Section 2) and the energy fluxes through surfaces far away (Section 3) and close to the interface (Section 4), we study the time-domain response, and the dependence of the energy fluxes (i.e., magnitude, spectral density and directivity) on the contrast in material parameters of the layers and on the velocity of the load (Section 5).

2. Model and solution

2.1. Model

We consider two coupled homogeneous isotropic elastic layers subjected to a surface load F that moves over the vertical layer interface, as shown in Fig. 1. The semi-infinite elastic layers are fixed to a rigid bottom, and their behavior is linear (i.e., small strains). Though we restrict the model to the two dimensions x and z (i.e., plane strain condition), it allows for the existence of body and free-surface waves and is thus appropriate for providing new insights into the mechanism of transition radiation. The velocity of the load is taken sub-critical (or: sub-sonic) and constant so that the transition radiation does not interfere with other possible radiation effects such as Mach/Vavilov–Cherenkov radiation (Ginzburg, 1996); however, the method of solution described in this paper is not restricted to this assumption. The assumption of constant load velocity implies that we disregard transient effects associated with acceleration/deceleration of the load.

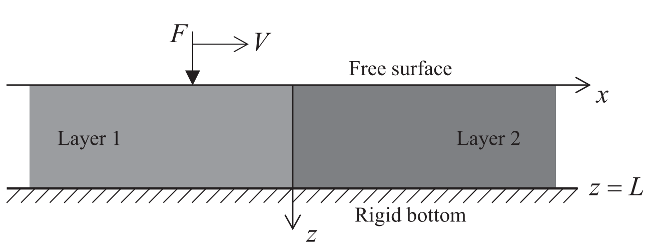

Fig. 1. A surface load F moving over the interface of two semi-infinite homogeneous isotropic elastic layers at constant velocity V. The layers are assumed to be in plain-strain condition, and their behavior is assumed linear (i.e., small strains).

The behavior of the layers is described by the equations of a classical elastic continuum (de Hoop, 1995; Aki and Richards, 2002; Achenbach, 2003). The boundary conditions for the stress $\boldsymbol{\sigma}$ and the displacement $\mathbf{u}$ at the horizontal edges are

$$\sigma_{zz}^{(1)}(x, z=0, t) = \begin{cases} -F\delta(x - Vt), & t < 0, \\ 0, & t \geqslant 0, \end{cases} \tag{1}$$

$$\sigma_{zz}^{(2)}(x, z=0, t) = \begin{cases} 0, & t < 0, \\ -F\delta(x - Vt), & t \geqslant 0, \end{cases} \tag{2}$$

$$\sigma_{zx}^{(i)}(x, z=0, t) = 0, \tag{3}$$

$$u_x^{(i)}(x, z=L, t) = 0, \tag{4}$$

$$u_z^{(i)}(x, z=L, t) = 0, \tag{5}$$

where the superscript $i = \{1, 2\}$ denotes the left or the right layer, respectively; L denotes the thickness of the layer (see Fig. 1), t denotes time, $\delta(\ldots)$ the Dirac delta function, and V is the velocity of the load. At the interface between the two layers, the following continuity conditions hold:

$$\sigma_{xx}^{(1)}(x=0, z, t) = \sigma_{xx}^{(2)}(x=0, z, t), \tag{6}$$

$$\sigma_{xz}^{(1)}(x=0, z, t) = \sigma_{xz}^{(2)}(x=0, z, t), \tag{7}$$

$$u_x^{(1)}(x=0, z, t) = u_x^{(2)}(x=0, z, t), \tag{8}$$

$$u_z^{(1)}(x=0, z, t) = u_z^{(2)}(x=0, z, t). \tag{9}$$

Finally, we have the radiation condition and the requirement that the response is finite at infinite distance away from the load as well as from the layer interface (where waves are excited, as discussed hereafter).

A semi-analytical solution to this problem can be found by splitting the solution into a steady-state part and a transient part for each of the domains ($x < 0$ and $x > 0$). The steady-state part is the so-called eigenfield of the load that, in the reference system that moves with the load, is stationary provided that the medium is translation-invariant along the line of load movement (Vesnitskii and Metrikin, 1996). The eigenfield is confined to the vicinity of the load due to its sub-critical velocity, and a modified eigenfield will be formed after the load has crossed the interface at $x = 0$; the difference in eigenfields can be considered as the source of transition radiation. The transient part of the solution captures the corresponding radiation field that, though excited by the load, propagates independently; hence, the transient part is referred to as the free field (throughout the paper, we refer to eigenfields and free fields when explicitly referring to the solutions in both layers). We thus search for the solution in the following form:

$$\mathbf{u}^{(i)}(x, z, t) = \mathbf{u}^{(i),\mathrm{e}}(x, z, t) + \mathbf{u}^{(i),\mathrm{f}}(x, z, t), \tag{10}$$

$$\boldsymbol{\sigma}^{(i)}(x, z, t) = \boldsymbol{\sigma}^{(i),\mathrm{e}}(x, z, t) + \boldsymbol{\sigma}^{(i),\mathrm{f}}(x, z, t), \tag{11}$$

where the superscripts “e” and “f” indicate the eigenfield and the free field, respectively. For ease of analysis, we apply the Helmholtz decomposition: