International Journal of Solids and Structures 77 (2015) 65–73

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

# Nonlinear wave propagation in a hexagonally packed granular channel under rotational dynamics

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ARTICLE INFO

*Article history:*
Received 4 April 2015
Received in revised form 2 July 2015
Available online 26 July 2015

*Keywords:*
Granular crystals
Hexagonal packing
Nonlinear dynamics
Solitary waves

ABSTRACT

In the present work, we numerically and experimentally investigate the propagation of nonlinear waves in a hexagonally packed granular channel. Specifically, we assemble a simple 1-2-1 granular chain, which triggers not only normal but also tangential interactions of particles up on a striker impact. We experimentally measure the transmission of nonlinear waves propagating along the channel direction using an embedded piezoelectric sensor particle. For numerical simulations, we introduce a discrete element model that accounts for both elastic and damping effects in axial and rotational directions. As a result, we find that the wave propagation in this 1-2-1 hexagonal architecture is governed strongly by the rotational dynamics of particles. We also verify that the effect of rotational damping is crucial for the accurate description of the particle's dynamics. The findings in this study hints the significance of particles' rotational dynamics when stress waves propagate along hexagonally packed particle channels even in highly ordered 2D/3D granular architectures.



## 1. Introduction

Granular crystals defined as a conglomerate of tightly packed and ordered granules have attracted significant attention from the scientific community due to their capability in forming and transmitting a wide range of stress waves (Nesterenko, 2001; Sen et al., 2008; Hladky-Hennion and de Billy, 2007; Herbold et al., 2009; Boechler et al., 2011; Shukla et al., 1993; Jayaprakash et al., 2011). When the particles interact with each other under the linear relationship (i.e., Hooke's law), they can form and propagate dispersive linear elastic waves through the periodic architectures of granules. These linear responses can be characterized by frequency band structures with distinctive pass- and stop-bands (i.e., band-gaps) (Hladky-Hennion and de Billy, 2007; Herbold et al., 2009; Boechler et al., 2011). If the interplay of the constituent particles is nonlinear, the granular crystals support the propagation of nonlinear waves. An example is a one-dimensional (1D) granular chain composed of spherical particles, whose interactions are governed by the classical Hertzian law (i.e., $F_m \sim \delta^{3/2}$, where $F_m$ is the compressive force and $\delta$ is the distance between the particles). Upon strong excitations, these granular chains can form and propagate highly nonlinear waves in the form of solitary waves (Nesterenko, 2001; Sen et al., 2008; Shukla et al., 1993; Jayaprakash et al., 2011; Kevrekidis, 2011). Compared to linear elastic waves, these nonlinear waves provide unique physical properties, such as high stability, near compact-supportedness, and controllability over their shape and propagation speed. By leveraging such characteristics, previous studies have proposed the usage of granular crystals as novel engineering devices, e.g., acoustic imaging devices (Spadoni and Daraio, 2010), impact mitigation layers (Hong, 2005; Doney and Sen, 2006), and acoustic switches (Li et al., 2014).

Despite the growing amount of research, the studies on nonlinear wave propagation in two-dimensional (2D) granular crystals have been largely unexplored. This is attributed to the complicated granular mechanics in 2D packing that inevitably involves not only axial, but also rotational interactions among frictional particles. A limited amount of work has been reported on the wave propagation in 2D granular architectures. For example, Sen et al. performed numerical studies on nonlinear wave propagation in hexagonally packed granular columns (Sen and Sinkovits, 1996). Likewise, Vitelli et al. studied energy transport in jammed 2D granular packings (Vitelli et al., 2010). Recently, researchers investigated oblique entrance of solitary waves to a 2D hexagonal lattice interface, revealing a refraction and reflection law in a form similar to the Snell's law (Tichler et al., 2013). However, these studies were based on numerical models without accounting for the rotational

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http://dx.doi.org/10.1016/j.ijsolstr.2015.07.017
0020-7683/

mechanics of granules. Consequently, most studies remained solely as numerical efforts without appropriate experimental validations. More recently, Leonard et al. explored numerically and experimentally versatile shapes of wave fronts in 2D granular crystals, but the focus was placed on the particles' axial interactions, neglecting their rotational dynamics (Leonard and Daraio, 2012).

We find a few discrete element models (DEMs) that took into account the effects of particle rotations based on the Hertz–Mindlin contact law (Mindlin, 1949; Cundall and Strack, 1979; Johnson, 1985). Velicky et al. used an effective medium approximation to describe mean-field of 2D lattices and to derive dispersion relationship (Velick and Caroli, 2002). Nishida et al. simulated the projectile impacting on a 2D granular bed made of polymers using 2D DEM (Nishida and Tanaka, 2010). However, the central theme of these works was not nonlinear wave propagation in closely-packed granular lattices, but linear wave dispersion or macroscopic granular flows. The mechanics of nonlinear wave propagation in 2D granular media under rotational dynamics may present totally different physical phenomena. For example, Jia *et al.* showed empirically that nonlinear wave propagation in disordered granules exhibits $V_s \sim F_m^{1/4}$ under the influence of particle jamming and rotations ($V_s$ and $F_m$ are wave speed and magnitude, respectively) (Jia et al., 1999). This is different from what we observe in aligned granular systems under dominant axial interactions (e.g., 1D granular chain or squarely packed 2D granular arrays), which exhibit $V_s \sim F_m^{1/6}$ (Nesterenko, 2001; Sen et al., 2008).

In this study, we simulate and measure the formation and propagation of nonlinear waves in ordered 2D granular systems. To investigate the fundamental physics of nonlinear wave propagation, we assemble a 2D granular lattice contained in a narrow channel, specifically in a 1-2-1 array. This is a simple setup that is designed to trigger both axial and rotational interactions of particles by impacting the system in a direction slanted from the axial alignments of particles. Previous studies have shown that if ordered granular systems are excited along the direction of particles' axial alignments, a DEM approach yield reasonably accurate results even without the consideration of rotational dynamics (Leonard and Daraio, 2012; Zhang et al., 2015). In this study, we excite the 1-2-1 channel by using a striker impact and record the propagation and attenuation of waves using a custom-made sensor particle. To verify the experimental results, we develop an in-house DEM that simulates the axial and rotational dynamics of tightly-packed, frictional particles. In particular, we introduce dissipative model of axial and tangential granular interactions using Tsuji's model (Tsuji et al., 1992).

Despite the highly ordered architectures of hexagonally packed granules assembled in this study, we find that rotational dynamics of particles plays a crucial role in the dissipative and dispersive trends of nonlinear wave propagation. Based on the damping factors obtained from experiments, the kinetic and potential energy profiles of the granular system are also calculated, and their temporal histories are reconstructed. The attenuation trend of the energy distribution demonstrates that a large portion of initial energy carried by the 1-2-1 channel is lost by rotational friction. After proper consideration of such rotational dynamics of particles, we observe that the numerical results of the dispersion and attenuation of nonlinear waves are in agreement with the experimental measurements. The findings in this study provide an empirical and computational proof that rotational dynamics is crucial in capturing nonlinear wave propagation even in a highly ordered granular system. We also observe that the nonlinear waves in this 1-2-1 system exhibit waveforms that do not show self-similar patterns during propagation, which is distinct from those formed in 1D granular chains. However, we find that the speed of propagating waves in this 1-2-1 system is governed strongly by their force amplitude in a pattern similar to the power-law observed in solitary waves propagating in 1D granular chains.

The rest of the manuscript is composed as below: In Section 2, we describe the experimental setup to investigate the attenuation and dispersion of nonlinear waves due to axial and rotational dynamics. We introduce the DEM based on Tsuji's model in Section 3 for numerical simulations. Section 4 includes the numerical and experimental results accompanied with discussion. We finish this manuscript with conclusions and future works in Section 5.

## 2. Experimental setup

We conduct two sets of experiments in this study. First, we assemble and test a straight granular chain as a reference case, where we assess the attenuation of solitary waves primarily due to the axial interactions of particles. The granular chain is composed of $N = 64$ chrome steel spheres (Bearing-quality E52100 alloy steel, McMaster) with radius $R = 9.525 \pm 0.0254$ mm, elastic modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.27$, and mass $m = 28.2$ g. The beads are placed on a low-friction polycarbonate plate and constrained by hardened and surface-finished stainless steel blocks to restrict the lateral motions of particles (Fig. 1(a)). The material properties of these stainless steel blocks are assumed to be identical to those of the chrome steel beads. We ensure there exists clearance between the granular chain and the side walls, so that the beads can move freely in the channel. The clearance is measured to be less than 0.051 mm by using a gap gauge.

An identical bead is used as a striker to generate a single pulse of solitary waves by impacting the front end of the granular chain. The striker is released from a ramp (see the left inset of Fig. 1(a)) with a solenoid, and the impact velocity is 0.55 m/s. We measure the propagation of solitary waves by embedding sensor particles in the granular chain (right inset of Fig. 1(a)). This instrumented particle contains a thin piezoelectric disc between two half spheres, which converts transmitted stress waves into voltage signals. The sensor bead is designed carefully to have the identical contact stiffness and mass compared to that of the regular particle in the chain. The calibration process of the sensor is described in Daraio et al. (2005). In this study, we place the first sensor particle at the beginning of the chain, which is directly impacted by the striker. This sensor plays a role of triggering the external data acquisition system up on the moment of impact. The second sensor bead is positioned along the chain, whose position is altered in every three-particle spot. The measured sensor signals are synchronized with respect to the impact moment (captured by the first sensor signal mentioned above) and are later reconstructed to describe the attenuation trends of solitary waves along the 1D chain. In each particle spot, we repeat five measurements for statistical treatments.

After evaluating the *axial* dissipative effect in a straight granular chain, we build and test hexagonally packed granular chains, where *rotational* dynamics start to affect the mechanism of nonlinear wave propagation. For the sake of simplicity, we assemble an ordered 1-2-1 granular array which is contained in a narrow channel as shown in Fig. 1(b). For the stability of the setup, the end of the particles' array consists of a pair of particles, while the beginning of the chain is a single particle, which is directly impacted by a striker. The total number of particles used in the chain is 36, composing 12 rows of single and double particle arrays. In the 1-2-1 setup, the width of the channel is slightly larger than two particles' diameters, such that any particle pairs can barely pass through the channel without being clamped. Before each