International Journal of Solids and Structures 77 (2015) 74–85

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Energy dissipation in the course of the fatigue degradation: Mathematical derivation and experimental quantification

Ali Kahirdeh, M.M. Khonsari *

Department of Mechanical Engineering, Louisiana State University, Baton Rouge, LA 70803, USA

ARTICLE INFO

Article history:
Received 12 December 2014
Received in revised form 27 May 2015
Available online 8 July 2015

Keywords:
Energy dissipation
Acoustic emission
Fatigue
Continuum mechanics

ABSTRACT

This paper is concerned with a continuum formulation for irreversible energy dissipation that accounts for generated acoustic emissions during the loading of the materials. Within a thermodynamically consistent framework, the coupling between the mechanical, thermal and acoustic fields is formulated. The evolution of the dissipative energy is experimentally measured as the material is degraded. A statistically similar behavior is observed in different forms of the dissipated energy as the material degrades.

1. Introduction

Energy dissipation and entropy production are two metrics that describe the irreversible processes occurring in a material subjected to loading cycles (Basaran et al., 2003; Basaran and Nie, 2004; Basaran and Yan, 1998; Yao and Basaran, 2012). Bryant et al. (2008) show that associated entropy production is a measure of material's degradation. Basaran and Nie (2004) proposed entropy as a quantity that connects the behavior of the structure at dislocation level, *e.g.* initiation of the micro-cracks, with the macroscopic response of the structure. The underlying physics of the degradation in various applications such as tribological processes (Nosonovsky, 2010a,b), friction (Nosonovsky and Bhushan, 2009; Nosonovsky and Mortazavi, 2012), wear (Doelling et al., 2000; Ling et al., 2002; Nosonovsky and Bhushan, 2009) and the fatigue problem of the machinery components (Bhattacharya and Ellingwood, 1998) are reported to be correlated with dissipation of energy (Meneghetti, 2007; Meneghetti and Quaresimin, 2011) and entropy production (Naderi et al., 2010; Naderi and Khonsari, 2012).

Fatigue degradation is a cumulative irreversible process that occurs due to the internal friction, initiation and growth of micro-fractures that manifest themselves in different forms of dissipation mechanisms including mechanical hysteresis, heat, and acoustic emissions. While heat and entropy generation (Basaran and Nie, 2004; Basaran and Yan, 1998; Naderi et al., 2010) have been utilized by many researchers to characterize the rate of material degradation and evolution of fatigue damage, the role of acoustic emission (AE) is often neglected, and its application is typically limited to the detection of the failure with qualitative interpretation of the material response.

Acoustic emission represents the response of a material to the changes of the external conditions and are generated internally due to deformations, micro-cracks (Evans et al., 1974), dislocations (Heiple et al., 1987), debonding of inclusions (Ono and Yamamoto, 1981), phase transitions (Rosinberg and Vives, 2012; Shaira et al., 2008), recrystallizations, slipping, twining (Li and Enoki, 2012; Muránsky et al., 2010) and the interaction between the cracked surfaces. The AE method has been used extensively for detecting the dynamics of fracture in a broad range of the materials. However, the physics of the AE phenomenon is not well understood (Rosinberg and Vives, 2012; Scruby, 1987), and it is unclear that how acoustic emissions should be incorporated quantitatively into models of dynamic fractures (Carpinteri and Lacidogna, 2008; Huo and Müller, 1993; Minozzi et al., 2003). According to (Minozzi et al., 2003), one of the reasons for the inconsistencies in the results of the models and experiments is the lack of considerations of the acoustic waves in most models. Additionally, models that describe the acoustic emissions of the materials are not often based on continuum mechanics (Bosia et al., 2008). They are typically based on implementation of the approaches such as Fiber-Bundle models (Bosia et al., 2008; Hidalgo et al., 2001) or Random Fuse Networks (Reurings and Alava, 2005).

The premise of this study is that the deteriorations in the integrity of a cyclically loaded specimen are reflected in several forms of dissipative mechanisms that occur concurrently. We aim at measuring the signatures of such dissipation mechanisms

---

* Corresponding author. Tel.: +1 225 578 9192; fax: +1 225 578 5924.
E-mail address: Khonsari@me.lsu.edu (M.M. Khonsari).

http://dx.doi.org/10.1016/j.ijsolstr.2015.06.032
0020-7683/

simultaneously as they occur inside the material. The signatures of the thermal, mechanical and acoustic dissipation are quantified experimentally during the life span of the material. To obtain a mathematical understanding of the experimental observations, we also develop a continuum depiction of the irreversible processes of a solid medium including the acoustic emissions and heat generation utilizing irreversible thermodynamic principles.

2. Theoretical

We define a continuous medium of the volume V and the differential volume of dV with two distinct states called initial (in equilibrium) and the deformed configuration, respectively. The transformation from the initial configuration to the deformed conditions maps each points on the continuum to a new position. By comparing the distance between two points of the un-deformed body and that of the same point on deformed configuration, one can determine if the continuum is under tension or compression. Considering the solid medium of inelastic materials as a thermodynamic system, the input mechanical work expended due to deformation of the continuum is dissipated through several mechanisms.

In this paper, we assume that three main dissipation mechanisms are plastic energy, heat generation and acoustic emissions. Heat generation is due to inelastic deformation and internal friction within the material under strain. The generated heat dissipates from the body through the three heat transfer mechanisms such as conduction, convection and radiation.

There are several sources for the acoustic emissions at any state of the deformation, regardless of whether it is elastic or plastic. The acoustic emission sources are demonstrated in Fig. 1 on a typical stress–strain diagram of the material. It encompasses internal friction and elastic deformations in the elastic region of the loading, and microstructural changes, deformation and movement of the dislocations, voids and inclusions, material's phase transition and recrystallization in the non-linear and plastic region.

Propagation of the acoustic waves in the material causes time-varying deformations, or vibrations, that are transmitted from one particle to another in the continuum. Acoustic emissions are small-scale, traveling waves or localized vibrations that produce elastic stress and strain fields due to internal restoring forces of the solid medium. A traveling wave inside the solid medium can be regarded as an acoustic excitation that is transmitted by elastic forces between the neighboring particles of the medium (Auld, 1973). Such waves can be measured experimentally by attaching a piezoelectric sensor to the surface of the material.

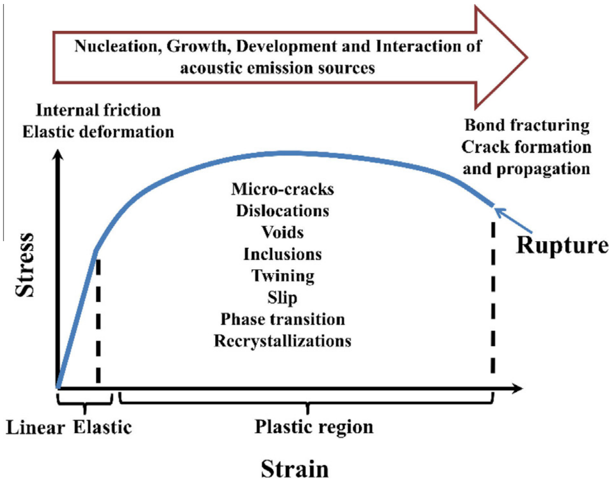

Fig. 1. Typical stress–strain behavior of the material and the acoustic emission sources at each stage.

We postulate that applying external load/deformation to a solid medium, excites the material in two ways. The first is the mechanical excitation that causes a stress and strain filed, and the second is due to acoustic emissions that propagates through the continuum. While the mechanical excitations can produce an elastic or plastic response in the material, acoustic excitations tend to be elastic in most solid materials. The propagation of acoustic emissions inside the solid medium can be described by elastic wave propagation (Auld, 1973). A region in space that is occupied by the solid is called *configuration* of the solid. In describing the motion of a solid, two configurations are often used. The first is the initial configuration known as *reference* and describes the configuration of an undeformed solid. The shape of the material changes under the external loads and occupies a new region at time instant of t resulting in a new configuration known as *deformed*. The continuum in the deformed configuration is depicted in Fig. 2 where both mechanical and acoustic stress fields are shown.

The mechanical field equations can be obtained by writing the governing equations for the continuum. Conservation laws of mass and momentum are required to obtain the equation of motion of the solid medium. Once the equation of the motion is derived, one can obtain the kinetic energy rate as in Eq. (1). See Groot and Mazur (1984), Malvern (1962) and Saouma (1998) for derivation:

$$\frac{\partial \rho\left(\frac{1}{2}v^2\right)}{\partial t} = -div\left(\rho\frac{1}{2}v^2\cdot v - \boldsymbol{\sigma}\cdot v\right) - \boldsymbol{\sigma}:\boldsymbol{D} + \rho\boldsymbol{F}\cdot v \qquad (1)$$

where ρ is the density, v is the velocity, $\boldsymbol{D}$ is the symmetric deformation tensor, $\boldsymbol{F}$ is the conservative body force. The term $\boldsymbol{\sigma}:\boldsymbol{D}$ is called the stress power which is the rate at which the medium is deforming.

2.1. The acoustic field equations

In this subsection the acoustic field equations and the energy transported by the acoustic waves through the solid medium are formulated. These equations are consistent with the work of Auld (1973) and Beltzer (1988). As mentioned in Section 2, acoustic sources of various nature induce waves that propagate through

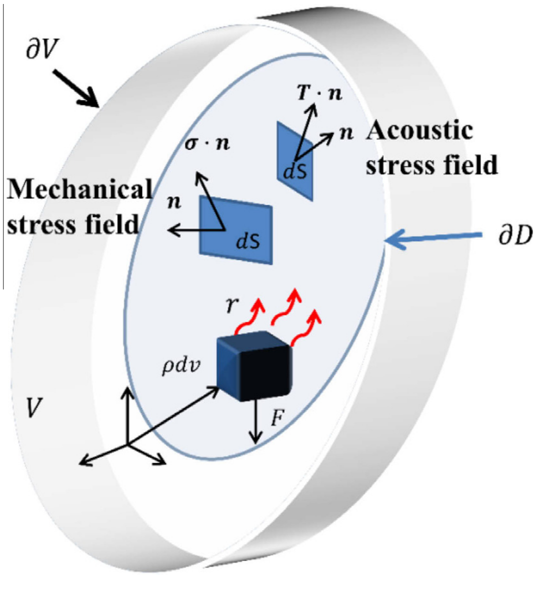

Fig. 2. The continuum in deformed configuration.