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A linear sampling method for detecting fluctuations of the wavefield in an elastic half-space

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ABSTRACT

A linear sampling method for an elastic half-space is developed to reconstruct fluctuations in the wavefield. The starting point of the formulation is the near-field equation that was also used by Baganas et al. (2006). Instead of examining the norm of the solution of the near-field equation, we define a solvability index in order to obtain the spatial distribution of the amplitude of the solvability index and thus describe the location of the fluctuations. A numerical method for the evaluation of the index is also provided for a simplified algorithm; this method is based on a projection theorem for a Hilbert space and a singular value decomposition. Numerical calculations were performed, and the results validated the efficiency of the proposed method for reconstructing the fluctuations of a wavefield.

1. Introduction

Inverse scattering analysis has a long history due to its inherent interest, as well as its applications in the fields of geophysical exploration, site characterization, medical imaging, nondestructive testing, and many other areas. Colton and Kress (1998) surveyed and reported a vast number of articles on inverse scattering analyses of acoustic and electromagnetic wave propagation. During the past ten years, many significant articles in this field have been published. For example, Guzina et al. (2003) used the regularized boundary integral equation method to solve the problem of mapping underground cavities. Pelekanos et al. (2004) presented a contrast source inversion method in a 2D elastic wavefield. Campman et al. (2006) formulated a method to estimate the wavefield that would have been measured if there were no near-receiver heterogeneities. Gélis et al. (2007) carried out a 2D full elastic waveform inversion using the Born and Rytov approximations. Romdhane et al. (2011) applied a 2D full waveform inversion to a shallow structure with complex topography. The authors' research group also presented a fast method for solving a volume integral equation (Touhei, 2009, 2011; Touhei et al., 2009) and applied it to an inverse scattering analysis (Touhei et al., 2014).

Among the various methods for the inverse scattering analysis, a linear sampling method, presented by Colton and Kirsch (1996), reconstructs the supports of the scatterers by tracing the norm of the solution of the far-field equation without information about the type of boundary conditions on the scatterers. A factorization method, presented by Kirsch (2011), reconstructs the support of scatterers by decomposing the far-field operator and examining its range, instead of solving the far-field equation. Colton and Kirsch (1996) used a linear sampling method with a 2D scalar Helmholtz equation for the far-field equation, but Fata and Guzina (2004) proposed a linear sampling method that uses the near-field equation. They provided the mathematical details of the near-field equation and then used it to analyze the reconstruction of cavities embedded in a 3D elastic half-space. Baganas et al. (2006) extended the method of the near-field equation to the inverse transmission problem of an elastic half-space. Guzina and Madyarov (2007) used a linear sampling method to reconstruct scatterers in piecewise-homogeneous domains. The authors' research group also used a linear sampling approach to evaluate the location and spatial spread of the fluctuations, and we ensured the accuracy of the reconstructed amplitudes by using the fast volume integral equation method (Touhei et al., 2014). In that paper, we presented only a brief outline of our method and its results, and a detailed description of the method and numerical results were left as an area of future work.

The purpose of the present article is to provide a detailed mathematical description of our method for evaluating the location and spatial spread of fluctuations in an elastic half-space and to provide numerical examples. The inversion equation used in this article is

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the near-field equation, which was also used by Fata and Guzina (2004) and Baganas et al. (2006). Instead of using the divergence properties of the near-field equation, we developed a solvability index for the equation; the spatial distribution of the fluctuations corresponds to the spatial distribution of the amplitude of the solvability index.

The organization of this paper is as follows.

In order to develop the concept of the solvability index of the near-field equation, we review the basic results of Fata and Guzina (2004) and Baganas et al. (2006), and we employ a method for the factorization of the operator (Kirsch, 2011). Using these results, we present the mathematical properties of the solvability index. Next, we present a method for evaluating the solvability index; this method is based on a projection theorem for a Hilbert space. After providing the formulation for the inverse scattering analysis, several numerical examples are presented to show the accuracy of this method. The intended application of the present method is to detect the spatial spreads of localized fluctuations in homogeneous background structures with high-velocity waves; an example of this is S waves with a velocity of 1 km/s.

2. Theoretical Formulation

2.1. Definition of the problem and the basic equation

Fig. 1 shows the wave problem defined in this article. The wavefield is a 3D elastic half-space in which there are fluctuations in contrast to a homogeneous background structure. On the free surface of the wavefield, there are both source and observation surfaces, which are denoted by Γ_1 and Γ_2, respectively. Distributed loads are applied to Γ_1 so that incident waves are scattered, and these are observed at Γ_2. The problem defined in this article is as follows:

Definition of the problem *We consider using information from the distributed loads at* Γ_1 *and the observed scattered waves at* Γ_2 *to reconstruct the spatial spread and the location of the fluctuations.*

As shown in Fig. 1, a Cartesian coordinate system is employed to express the wavefield; the vertical axis is denoted by x_3. A spatial point in the wavefield is expressed as

$$x = (x_1, x_2, x_3) \in \mathbb{R}^2 \times \mathbb{R}_+ = \mathbb{R}^3_+ \tag{1}$$

where the subscript index indicates the component of the Cartesian coordinate system. The free boundary of the elastic half-space, denoted by B, is at $x_3 = 0$. In the following, the summation convention is applied to the subscript indexes describing the Cartesian coordinate system. Using the summation convention, the scalar product of wavefunctions is defined as follows:

$$(\varphi_i,\ \psi_i)_{L_2(\Gamma_k)} = \int_{\Gamma_k} \varphi_i^*(x)\psi_i(x)\mathrm{d}\Gamma_k(x), \quad \varphi_i, \psi_i \in L_2(\Gamma_k) \tag{2}$$

where k takes 1 or 2. The L_2 norm is defined as the scalar product, which is represented in the following form:

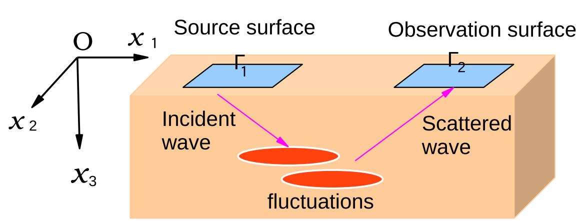

Fig. 1. Wave problem considered in this article.

$$\|\varphi_i\|^2_{L_2(\Gamma_k)} = (\varphi_i,\ \varphi_i)_{L_2(\Gamma_k)} \tag{3}$$

The Lamé constants and the mass density that characterize the wavefield are expressed as

$$\begin{aligned} \lambda(x) &= \lambda_0 + \tilde{\lambda}(x) \\ \mu(x) &= \mu_0 + \tilde{\mu}(x) \\ \rho(x) &= \rho_0 + \tilde{\rho}(x), \quad (x \in \mathbb{R}^3_+) \end{aligned} \tag{4}$$

where λ_0, μ_0 are the background Lamé constants, and ρ_0 is the mass density. Their respective fluctuations are $\tilde{\lambda}$, $\tilde{\mu}$, and $\tilde{\rho}$. The fluctuations are assumed to be characterized as

$$\tilde{\lambda}(x),\ \tilde{\mu}(x),\ \tilde{\rho}(x) \in C^1_0(\mathbb{R}^3_+) \tag{5}$$

We define the support of the fluctuations E such that

$$E = \text{supp}\ \tilde{\lambda}(x) \cup \text{supp}\ \tilde{\mu}(x) \cup \text{supp}\ \tilde{\rho}(x) \tag{6}$$

and we let ∂E express the boundary of E. Namely, E can be expressed by

$$E = E^o \cup \partial E \tag{7}$$

where E^o is the set of internal points of E. In addition, we assume that the complementary set of E, denoted by E^c, is connected. In addition, the free boundary B and the fluctuated regions are disjoint:

$$E \cap B = \emptyset \tag{8}$$

The time dependency is assumed to be $\exp(i\omega t)$, where ω is the circular frequency, and t is the time. Based on the time dependency, the governing equation and boundary condition for the wavefield are

$$\begin{aligned} &L_{ij}(\partial)w_j(x) = -N_{ij}(\partial, x)w_j(x) \\ &n_j(x)T_{ijk}(\partial)w_k(x) = \begin{cases} \tau_i(x) & x \in \Gamma_1 \subset B \\ 0 & x \in B \setminus \Gamma_1 \end{cases} \end{aligned} \tag{9}$$

where w_j is the displacement field (total wavefield), τ_i is the distributed load at Γ_1, and L_{ij}, N_{ij}, and T_{ijk} are the following differential operators:

$$L_{ij}(\partial) = (\lambda_0 + \mu_0)\partial_i\partial_j + \delta_{ij}\mu_0\partial_k\partial_k + \delta_{ij}\rho_0\omega^2 \tag{10}$$

$$\begin{aligned} N_{ij}(\partial, x) = &\left(\tilde{\lambda}(x) + \tilde{\mu}(x)\right)\partial_i\partial_j + \delta_{ij}\tilde{\mu}(x)\partial_k\partial_k + \left(\partial_i\tilde{\lambda}(x)\right)\partial_j \\ &+ \delta_{ij}(\partial_k\tilde{\mu}(x))\partial_k + \left(\partial_j\tilde{\mu}(x)\right)\partial_i + \delta_{ij}\tilde{\rho}(x)\omega^2 \end{aligned} \tag{11}$$

$$T_{ijk}(\partial, x) = \mu(x)\delta_{ik}\partial_j + \mu(x)\delta_{jk}\partial_i + \lambda(x)\delta_{ij}\partial_k \tag{12}$$

Note that δ_{ij} is the Kronecker delta, ∂_j is the partial differential operator, $n_j(x)$ is the normal vector of the boundary at the point x, and the subscript indicates the component of the coordinate system. The Green's function for the background structure of the wavefield is important in the formulation, as well as in the numerical calculations; this function is defined as

$$\begin{aligned} &L_{ij}(\partial_x)G_{jk}(x, y) = -\delta_{ik}\delta(x - y) \\ &n_j(x)T^{(0)}_{ijk}(\partial_x)G_{kl}(x, y) = 0, \quad (x \in B) \end{aligned} \tag{13}$$

where $x,\ y \in \mathbb{R}^3_+$ are the field and source points for the Green's function, ∂_x denotes the differential operator for the field point, $\delta(\cdot)$ is the Dirac delta function, and $G_{ij}(\cdot,\cdot)$ is the Green's function. Note that $T^{(0)}_{ijk}$ is the differential operator defined by

$$T^{(0)}_{ijk}(\partial) = \mu_0\delta_{ik}\partial_j + \mu_0\delta_{jk}\partial_i + \lambda_0\delta_{ij}\partial_k \tag{14}$$

We will use the spectral form of the Green's function (Touhei, 2009) for the numerical examples in this article; this is given by