International Journal of Solids and Structures 72 (2015) 108–117

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Gradient-elasticity for honeycomb materials: Validation and identification from full-field measurements

Julien Réthoré [a,*], Christine Kaltenbrunner [a], Thi Bach Tuyet Dang [a], Philippe Chaudet [a], Manuel Kuhn [b]

[a] LaMCoS, Université de Lyon/INSA Lyon/CNRS UMR 5259, Bat. Jacquard, 27 Avenue Jean Capelle, F-69621 Villeurbanne, Cedex, France
[b] Mechanical Engineering Department, INSA Lyon, Bat. Jacquard, 27 Avenue Jean Capelle, F-69621 Villeurbanne, Cedex, France

ARTICLE INFO

Article history:
Received 31 March 2015
Received in revised form 12 June 2015
Available online 22 July 2015

Keywords:
Gradient-elasticity
Digital image correlation
Identification

ABSTRACT

Gradient-elasticity and more generally gradient-enhanced continuum models have been extensively developed since the beginning of the twentieth century. These models have shown the ability to account for the effect of the underlying material heterogeneity at the macroscopic scale of the continuum. Despite of a great theoretical interest, gradient-enhanced models are usually difficult to interpret physically and even more to identify experimentally. This paper proposes an attempt to validate and identify from experimental data, a gradient-elasticity model for a material with a periodic micro-structure. A set of dedicated experimental and numerical tools are developed for this purpose: first, the design of an experiment, then two-scale displacement field measurements by digital image correlation with dedicated post-processing techniques and finally a model updating technique. This paper ends up with the full set of first and second-order elastic constants of a gradient-elastic model which macroscopic kinematic has been validated by investigating the deformation of the unit cells at the microscopic scale.

1. Introduction

Gradient-elasticity and more generally gradient-enhanced continuum models have been extensively developed since the pioneering work by Cosserat et al. (1909), Mindlin (1964), Toupin (1964), and Eringen and Suhubi (1964). These models have shown a number of useful properties for e.g. capturing size effects. The main purpose of these models is to incorporate the effect of the underlying heterogeneity of materials at the macroscopic scale of the continuum. Despite of a great theoretical interest, gradient-enhanced models are usually not easy to interpret physically. They are also difficult to implement numerically because they lead to higher order partial differential equations that require higher continuity of the displacement description. Furthermore, these models involve numerous material constants, the full set of these constants being almost impossible to obtain experimentally. Among the reasons for the gradient-enhanced material parameters to be difficult to identify experimentally, is their localized influence (where intense strain variations occur) and the low amplitude of their contribution to the global response of a material. To circumvent these experimental difficulties, a great effort has been made by researchers to develop accurate homogenization techniques of higher order. Among the proposed methodologies, one could mention the work by Kouznetsova et al. (2002) and Forest (1998) or Gologanu et al. (1995). However, as evidenced by Forest and Trinh (2011), the hypothesis of these homogenization techniques concerning the deformation of the control volume is not clear and the estimation of effective properties of a heterogeneous material is therefore subjected to some limitations.

The recent development of full-field displacement measurement techniques by digital image correlation allowing for a strong coupling between experiments and numerical simulations, e.g. Besnard et al. (2006), Réthoré et al. (2008, 2013), opens new perspectives for studying experimentally gradient-enhanced models. Indeed, they allow for a local investigation of the kinematic field during the experiment and, there is thus a potential for validating and/or identifying the material constants of gradient-enhanced models. In Burteau et al. (2012), an attempt to measure the deformation of a cellular material at the cells' scale was proposed. However, the deformation parameters are estimated from the analysis of the morphology of the cells before and after deformation, without quantifying the local deformation of the material at the microscopic scale. Using full-field measurements, this paper proposes a first attempt to validate and identify a gradient-elasticity model for a material with a periodic micro-structure. A set of dedicated experimental and numerical tools are developed for this purpose. First, the design of an experiment allowing for activating

* Corresponding author.
E-mail address: julien.rethore@insa-lyon.fr (J. Réthoré).

http://dx.doi.org/10.1016/j.ijsolstr.2015.07.015
0020-7683/

gradient-related phenomena in a model material is presented in Section 2. Then, in Section 3, the validation of a strain-based micro-morphic kinematic is proposed using the displacement fields measured at both the scale of the material micro-structure and the scale of the analyzed structure. Last, Section 4 is dedicated to the identification of the macroscopic continuum derived from the experimentally validated micro–macro kinematic relationship.

2. Experimental setup

2.1. Sample

To investigate experimentally the variations of the displacement in a heterogeneous media at different scales, a special setup has been designed. This setup is based on a specimen loaded in tension. The loading device is standard but the specimen is obtained from a 3D printer. As shown in Fig. 1, it has a central squared shaped part with an honeycomb structure (periodic tilling of hexagonal unit cells) whereas the ends of the sample are made homogeneous in order to be clenched by the grips. The global coordinate frame ($\underline{\mathbf{e}}_1, \underline{\mathbf{e}}_2$) is defined in Fig. 1 as the horizontal and vertical axis. The honeycomb cell's edge is 0.68 mm and the cell wall thickness is 0.29 mm. The overall dimension of the central part of the specimen is 90 mm. Its thickness is 10 mm. The honeycomb structure is thus made of 67×67 cells. A row of 15 cells with an angle of 30°, with respect to the loading direction ($\underline{\mathbf{e}}_1$), has been removed in the center of the specimen to create a strongly varying multi-axial strain state.

2.2. Digital image correlation

A black and white speckle pattern has been applied on the specimen surface using spraid paint. During the test, a digital camera records images of the specimen surface with a definition of 6576×4384 pixels to allow for displacement field measurements using digital image correlation. The camera is mounted with a 200 mm lens leading to a conversion factor from pixel to meter of 19 μm per pixel. Using this setup, the distance between two unit cell centers along the horizontal axis is 66 pixels. Digital image correlation is based on the local conservation of the gray levels between two images f and g:

$$f(\underline{\mathbf{X}}_p) = g(\underline{\mathbf{X}}_p + \underline{\mathbf{d}}(\underline{\mathbf{X}}_p)), \quad (1)$$

$\underline{\mathbf{X}}_p$ giving the position of a pixel in the global coordinate system and $\underline{\mathbf{d}}$ being the unknown measured displacement. A non-linear least-squares resolution scheme of this equation is adopted in the following Besnard et al. (2006). This allows for using, as in the following, a finite element mesh to define the unknown kinematic. Minor modifications in the implementation of the technique compared to what was proposed by Besnard et al. (2006) have been adopted. Mainly, the image gradient is calculated using a central finite difference scheme and the sub-pixel gray level interpolation is cubic-spline. Here, the reference image f, shown in Fig. 1, is recorded with zero force and g is the image recorded just before specimen failure.

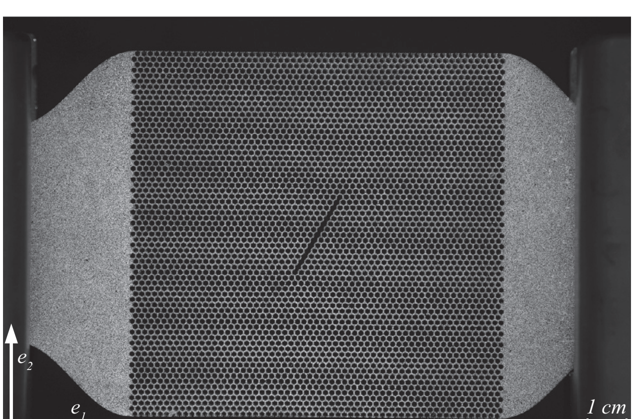

Fig. 1. Image of the sample in the tensile device. The image has a definition of 6576×4384 pixels.

Using this setup, first, the local and global variations of the measured displacement field are analyzed in Section 3. Then, the effective properties of the macroscopic continuum are identified in Section 4.2.

3. Two-scale displacement field analysis

3.1. Theoretical background

The displacement field of a heterogeneous media is usually considered at two separate scales. From a material point at the macroscopic scale defined by its coordinates $\underline{\mathbf{X}}$, a local microscopic volume is described through the local coordinates $\underline{\mathbf{x}} = \varepsilon \underline{\mathbf{X}}$. This change of scale is defined by the small parameter ε set as the ratio between the characteristic size of the control volume l and the characteristic size of the macro-scale L. In this context, the local fluctuations of the displacement at the microscopic scale $\underline{\mathbf{u}}(\underline{\mathbf{X}}, \underline{\mathbf{x}})$ are expanded around a given material point X:

$$\underline{\mathbf{u}}(\underline{\mathbf{X}}, \underline{\mathbf{x}}) = \underline{\mathbf{u}}_0(\underline{\mathbf{X}}, \underline{\mathbf{x}}) + \varepsilon \underline{\mathbf{u}}_1(\underline{\mathbf{X}}, \underline{\mathbf{x}}) + \varepsilon^2 \underline{\mathbf{u}}_2(\underline{\mathbf{X}}, \underline{\mathbf{x}}) + \dots \quad (2)$$

Eq. (2) is then transformed following Boutin (1996) to set the local displacement $\underline{\mathbf{u}}$ as a combination of local boundary value problems depending on the macroscopic kinematic variables $\underline{\mathbf{U}}(\underline{\mathbf{X}})$ and its gradients:

$$\underline{\mathbf{u}}(\underline{\mathbf{X}}, \underline{\mathbf{x}}) = \underline{\mathbf{U}}(\underline{\mathbf{X}}) + \varepsilon \underline{\mathbf{L}}_1(\underline{\mathbf{x}}, \underline{\nabla}_{\underline{\mathbf{X}}} \underline{\mathbf{U}}(\underline{\mathbf{X}})) + \varepsilon^2 \underline{\mathbf{L}}_2(\underline{\mathbf{x}}, \underline{\nabla}_{\underline{\mathbf{X}}} \underline{\nabla}_{\underline{\mathbf{X}}} \underline{\mathbf{U}}(\underline{\mathbf{X}})) + \dots, \quad (3)$$

where $\underline{\mathbf{L}}_1$, $\underline{\mathbf{L}}_2$ are localization operators to be defined. These boundary value problems classically use the macroscopic kinematic variables to prescribe the local displacement fluctuations on the boundary of the control volume. As the expansion in Eq. (2) has been limited, intentionally, to order 2, a quadratic polynomial (Gologanu et al., 1995; Forest, 1998, e.g.) is subsequently used to prescribe the variation

$$\underline{\tilde{\mathbf{u}}}(\underline{\mathbf{x}}) = \underset{\sim}{\mathbf{F}} . (\underline{\mathbf{x}} - \underline{\mathbf{X}}) + \frac{1}{2} \underset{\approx}{\mathbf{D}} : ((\underline{\mathbf{x}} - \underline{\mathbf{X}}) \otimes (\underline{\mathbf{x}} - \underline{\mathbf{X}})) \quad (4)$$

of $\underline{\mathbf{u}}$ on the boundary of the control volume $V(\underline{\mathbf{X}})$. In this definition, $\underset{\sim}{\mathbf{F}}$, respectively $\underset{\approx}{\mathbf{D}}$, are kinematic parameters of the deformation of the unit cell. The operators. and: define simple and double contraction and $\otimes$ the tensorial product. In practice, Dirichlet boundary conditions as defined by $\underline{\tilde{\mathbf{u}}}$ reveal too stiff when estimating the effective properties of the heterogeneous media. Further, in the case of a material obtained by a periodic tilling of unit cells, a supplementary fluctuation $\underline{\mathbf{v}}$ is allowed:

$$(\underline{\mathbf{u}}(\underline{\mathbf{X}}, \underline{\mathbf{x}}) - \underline{\mathbf{U}}(\underline{\mathbf{X}}))_{\partial V(\underline{\mathbf{X}})} = \underline{\tilde{\mathbf{u}}}(\underline{\mathbf{x}}) + \underline{\mathbf{v}}(\underline{\mathbf{x}}). \quad (5)$$

This additional fluctuation is usually assumed to have a periodic behavior but it has been shown by Forest and Trinh (2011) that a non-periodic fluctuation may be obtained in practice. This results were obtained by analyzing full-field finite element simulations. The aim of this section is to investigate these conditions experimentally.