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A 3D exact analysis of the boundary layer effect of asymmetric piezoelectric laminates with electromechanical coupling

Zhangjian Wu [a,*], Chao Han [a], Zhongrong Niu [b]

[a] School of MACE, The University of Manchester, Manchester M13 9PL, UK
[b] Department of Engineering Mechanics, Hefei University of Technology, Hefei 230009, PR China

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A B S T R A C T

A hybrid analysis of the interfacial stresses near the free edges of piezoelectric unsymmetrical laminated plates under uniform extension is presented based on three-dimensional piezoelasticity. A state space equation for cross-ply asymmetric piezoelectric laminates is obtained by considering all the independent elastic and piezoelectric constants of the laminates. The equation satisfies the open-circuit electric, free boundary conditions at two opposite free edges. Using a transfer matrix and a recursive solution approach, a three-dimensional exact solution has been derived and this has been validated by comparing analytical results with those from finite element analyses. Discussion and comments on solutions obtained using the equations proposed are presented. Since all the continuity conditions across the interfaces between material layers are satisfied, the concentrations of interfacial stresses near free edge boundaries are determined precisely. The significant electromechanical influence of the boundary layer effect is found quantitatively.

1. Introduction

Composite laminates are used extensively in multifunctional structures (Ishii et al., 2005) including smart piezoelectric structures which can perform sensing, controlling, actuating with distinct direct and converse piezoelectric effects. Such structures have been used in applications such as structural vibration control, precision positioning, electric package and nanotechnology because of their large electromechanical coupling effect, wide bandwidth and quick response ability (Kapuria et al., 2010). They can be used in the detection and generation of sonar waves (Yang, 2005) and in ultrasonic transducers for medical imaging and signal capturing.

Piezoelectric structures are often made from multi-layered thin films of dissimilar materials in the forms of stacks. An example of the use of these structures is in the design and manufacture of the ultrasonic wave generators used in the "black-boxes" in commercial aircraft. A further application is in stealth military aircraft because of the ability of piezoelectric structures, with appropriate layers, to provide signal absorption and shielding (Ra'di et al., 2013). As an emerging application, the development of nanogenerators has demonstrated a possible requirement for multilayer piezoelectric structures in the design of the self-sufficient power source that draws energy directly from ambient mechanical resources (Wang, 2012). Most recently, researchers have engineered the piezoelectric effect into graphene which has the potential to provide the dynamic control of nanoscale electromechanical devices (Ong and Reed, 2012).

Material parameters, interfaces and/or edges in laminated structures raise concerns that they might affect the structural performance of structures and systems in which they are used (Shang et al., 2005). In particular, at the interface between different materials, material and geometric discontinuities cause stress concentrations near free edges, an effect known as the "free-edge effect" (Pipes and Pagano, 1970; Wang and Choi, 1982; Kapuria et al., 2010) or the boundary layer effect. Due to this kind of discontinuity of material properties at interfaces, often accompanied by a high concentrated 3D stress field, interfacial or interlaminar failures such as delamination, local buckling or matrix cracking may occur near free edges. For piezoelectric laminated structures, the intrinsic physical properties of piezoelectric materials can, in practice, also induce simultaneously electromechanical coupling effects. A precise structural analysis of the deformation and failure modes (Kapuria et al., 2010; Tan et al., 2009) of laminated piezoelectric structures is complicated but the analysis of the "free-edge" or boundary layer effect, especially the interlaminar

* Corresponding author. Tel.: +44 161 3065792.
E-mail address: jack.wu@manchester.ac.uk (Z. Wu).

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stresses at a free edge is of great importance in the design and performance control of piezoelectric composite laminates.

Over the last forty years, much effort has been dedicated to the formulation of appropriate lamination models that could cope with the peculiar displacement, strain and stress fields across the cross-section of a laminate. A significant amount of literature has been produced on this topic, as cited in many review articles concerning its application to composite plate/shell structures (Kapania, 1989; Reddy and Robbins, 1994; Mittelstedt and Becker, 2007; Zhang and Yang, 2009; D'Ottavio et al., 2013).

Among various theoretical lamination models, classical lamination theory (CLT) is the most popular (Lee, 1990). However, CLT assumes a linear distribution of displacement across the thickness of an entire laminate and neglects transverse shear deformation which induces significant inaccuracies in the stress analysis of piezoelectric composite laminates.

To evaluate the boundary layer effect in composite laminates (Pipes and Pagano, 1970; Wang and Choi, 1982; Becker, 1993; Tahani and Nosier, 2003; Yang, 2005; Zhang et al., 2006) with electromechanical coupling effect, Ye and He (2001) solved the problem of electric field concentrations of a pair of parallel electrodes arrayed in one plane by using Fourier transform to reduce the electro-elastic boundary value problem to the solutions of integral equations. Using a finite element (FE) model, Yang et al. (2006) assessed the influence of geometrical and material parameters on the interlaminar stresses in a piezoelectric laminated beam. Based on the similar formulation, Artel and Becker (2005) detected the influence of the electromechanical coupling on the interlaminar stresses and electric fields near the free edges of symmetric laminates under uniaxial extension. To delineate the electromechanical behaviour of Artel and Becker's model an analytical solution was developed using a Reddy's Layer-Wise Theory (LWT) (Reddy, 2004) by Mirzababaee and Tahani (2009). Mannini and Gaudenzi (2004) used a multi-layer higher-order finite element method (FEM) to investigate interlaminar stress concentrations between a laminate and distributed piezoelectric actuators near free ends. Employing the improved zigzag LWT, Kapuria and Kumari (2012) investigated the boundary layer effects in a cross-ply piezoelectric laminate with Levy-type boundary conditions.

Recently researchers seem to have accepted that LWTs with first order or higher-order shear deformations were more accurate in displacement/stress field-based theories (D'Ottavio et al., 2013; Huang and Kim, 2014). However, as with traditional 2D laminate theories, Reddy's LWT laminate theories assume that some displacement and electric potential variables are polynomial functions of the coordinate in thickness direction of the laminate. In CLT, transverse displacement is a z-independent function and in-plane displacements are linear functions of the z coordinate. Reissner's theory considers the influence of the first order shear deformation and Ambertsumyan's theory (Ambartsumyan, 1970) assumes quadric functions of z for the transverse stresses. Reddy's LWT laminate theory is actually an extension of Ambertsumyan's theory for laminates. D'Ottavio et al. (2013) showed that a refined LWT with a transverse displacement field varying in each layer according to a fourth-order polynomial could include free-edge effects. As mentioned earlier, there are inconsistencies (Wu and Wardenier, 1998) among the basic equations of elasticity if a solution is presented in the form of polynomial functions in a 3D exact analysis and the boundary conditions at free edges are generally fulfilled only in an average integral sense. LWTs can provide quasi-3D results (to be more exact 'quasi' should be replaced with 'incomplete') but their models may not always be able to capture free-edge effects efficiently due to inconsistent assumptions of displacement functions and due to a demand for a number of unknown variables.

Another approach to the analysis of laminated composite structures is to use stress function-based theory which is claimed to satisfy not only stress continuity but also traction free boundary conditions (Huang and Kim, 2014). This kind of theory appears to be attractive and applicable to the interlaminar stress analysis of piezoelectric composite laminates. The approach however adopts complementary energy principles with variational formulations (Yin, 1994; Yin, 1994). The proposed stress-based layer-wise polynomial can only satisfy the equilibrium in an integral sense (not for the stress state in any point) (Huang and Kim, 2014). By using complementary energy principles, the kinematics may be fulfilled in an integral sense. Results obtained using this approach are only approximations.

Although the FEM can usually provide approximate descriptions for displacements, in-plane stresses and electric variables in the analysis of the piezoelectric laminates, the predicted through-the-thickness stresses and electric variables cannot satisfy the continuity conditions at interfaces of the laminates (Sheng et al., 2007; Wu and Kamis, 2012) and these results become unreliable for a free-edge analysis. This will be further clarified at the end of this paper.

To the authors' knowledge, it is now well established that the simplest 2D models, such as CLT, can be effectively used for a stiffness design that accounts for the laminate's gross response. If a more accurate representation of the stress field is required, for instance for a strength design, refined models would need to be employed (D'Ottavio et al., 2013).

In this paper, the state space method (Wu and Wardenier, 1998) is adopted to investigate the free-edge effect and the electromechanical coupling effect in cross-ply piezoelectric asymmetric laminates. This was based on the pioneering work of Pipes and Pagano (1970) for a symmetric non-piezoelectric composite laminate since very few exact stress analyses for asymmetric piezoelectric laminates under a uniform axial extension were available. On the basis of 3D piezoelasticity (Yang, 2005), an analytical solution that fulfills both mechanical and electric boundary conditions is sought. Since the solution satisfies all the continuous fields, especially the interlaminar displacement, stress and electric displacement continuities between dissimilar material layers, it is an exact solution and can be used as a benchmark to assess the precision of various 2D piezoelasticity plate/shell theories and 3D numerical FEM simulations.

2. Formulation of the problem

The description for a general structure using the linear theory of piezoelectricity consists of the equations of motion and charge (Nye, 1985; Yang, 2005),

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i, \quad D_{i,i} = f_e \tag{1}$$

the strain–displacement and electric field-potential relationships,

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2, \quad E_i = -\varnothing_{,i} \tag{2}$$

and the constitutive relationships,

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \quad D_i = e_{ijk}\varepsilon_{jk} + \epsilon_{ij}E_j \tag{3}$$

$\boldsymbol{u}$ is the mechanical displacement vector, $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{\varepsilon}$ is the strain tensor, $\boldsymbol{E}$ is the electric field, $\boldsymbol{D}$ is the electric displacement vector, $\varnothing$ is the electric potential, ρ is the mass density, $\mathbf{f}$ is the body force per unit volume and f_e is the body free charge density. The coefficients c_{ijkl} , e_{kij} and ϵ_{ij} are the elastic, piezoelectric and dielectric constants.

As shown in Fig. 1, a rectangular piezoelectric unsymmetrical laminated plate is subjected to a constant axial deformation comprising a uniform strain ε_0 along x direction. The plate has a