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A direct minimization technique for finding minimum energy configurations for beam buckling and post-buckling problems with constraints

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ABSTRACT

We present a novel technique to simulate the deformation of a cantilever elastic beam constrained in a curved solid channel subject to end forces. We pose this as the minimization of an energy functional and solve it by a variant of a dynamic programming approach called the Viterbi algorithm. The core idea of this approach is to discretize the variables describing the potential energy and to construct a set of admissible configurations of the beam. The Viterbi algorithm is then employed to search through the set of possible beam configurations and locate the one with the minimum potential energy in a very computationally efficient way. The new approach does not require any gradient computations and could be considered as a direct search method, and thus can be guaranteed to find the global minimum potential energy. Also the constraints can be automatically satisfied by constructing the proper set of all the possible configurations. The approach can also be used to find feasible starting configurations associated with conventional minimizing algorithms.

1. Introduction

The work presented in this paper was originally motivated by the biomedical related problem pertaining to the deformation of tubes passing through the esophagus (or the like-organs) during endoscopic operations (Fig. 1). Generally speaking, this problem can be thought as a buckled beam problem (Chen and Li, 2007) leading us to consider the simplified problem of the buckling of a two-dimensional (2D) beam constraint between two parallel curved solid walls.

The post-buckling phenomenon of beams free of geometric constraints subjected to a combined load consisting of a uniformly distributed axial load and concentrated load at the free end has been solved by Lee (2001). Beams with various boundary conditions have been reported in the literature. For instance, Chai (1998) has shown the buckling configurations of a simply supported beam with the constraints of parallel straight solid walls by assuming point/line contact regions and computing the reaction forces in these regions to find the buckled shape.

Chen and Li (2007) and Lu and Chen (2008) investigated the 2D beam buckling problem of in a channel with symmetric boundary conditions. In their work, both the inside and outside channel walls were assumed to be circular. They first carried out a sequence of experiments and then identified different deformation patterns of the beam, i.e., segments of beams in contact with the channel walls. Based on the observed patterns, each segment of the beam was listed independently, and the corresponding differential equations together with boundary conditions were developed. The full solution resulted in combining all solutions associated with the different patterns. The technique by Chen et al. has the advantage to be straight forward and easy to solve 2D beam buckling problems in a circular channel with symmetric boundary conditions. However, the approach requires experimentations for each geometry and boundary conditions to identify deformation patterns, which can then be incorporated into the segments of the differential equation. This is because the solution method depends upon knowing the type of contacting and non-contacting regions in priori. This severely limits the application of the approach since each geometry and boundary conditions have to be reinvestigated.

On the contrary, the method we propose is based on energy minimization and allows to find regions of contact without the need to resort to experiments or assuming deformation patterns. The technique presented here extend the work by Doraiswamy

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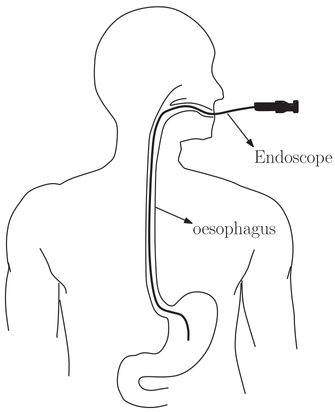


Fig. 1. The figure shows the endoscope passing through oesophagus.

et al. (2012) by introducing an "adaptive states quantization technique". To elaborate, in previous work the admissible positions of each node of beam can be determined without consideration of other nodes, since the possible vertical positions of each end of the beam are the same. Here, however due to the change in channel geometry, the possible location of any node depends upon the locations of the previous node. In other words the nodes locations are adaptively defined in order to satisfied the inextensible constraints. This reduced a substantially change in the search algorithm. A noteworthy features of the approach is that it can be used for any constraining geometry without the need for any experimentation. We show this by considering the example of a trapezoid curved channel constraint.

2. Qualitative experimental observations

Qualitative experiments have been used by Chen and Li (2007) and Lu and Chen (2008) to visualize the possible buckled shapes of a beam with symmetric boundary conditions (i.e. the two ends of the beam are either free to move or clamped) and constrained by a circular channel.

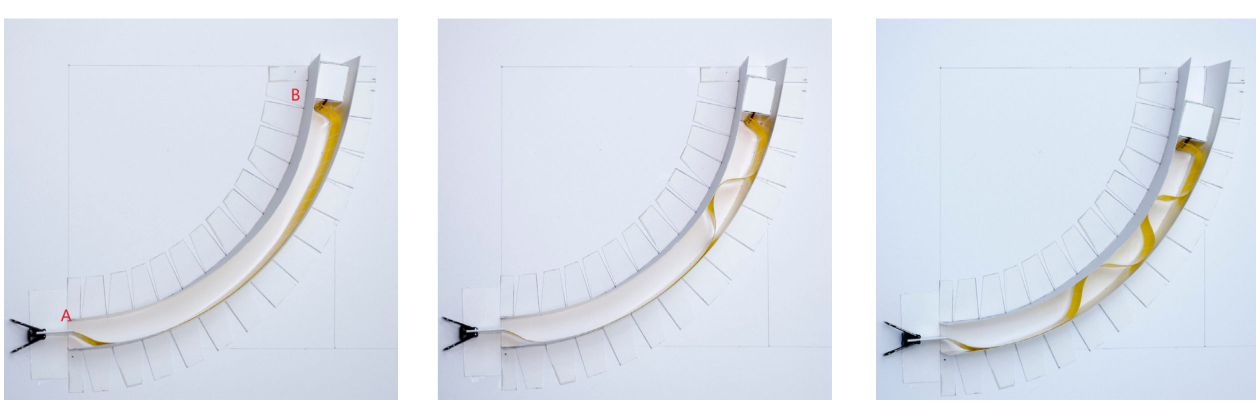
In contrast, our work is broader in the sense that we seek to study problems with non-symmetric boundary conditions such as those with one end clamped and one end pined. Therefore, the experimental results by Chen et al. cannot be used to serve as direct comparison with our simulations, which leads us to design our own experiments (see Fig. 2) to be used as benchmark.

In order to illustrate the technique based on Viterbi algorithm, we first consider the case of a circular channel (We stress that the technique is not limited to circular channels. Appendix A contains the generalization to non-circular channels). Fig. 2 depicts the set-up. Poster-board type paper is bent to form a quarter circular channel with outer radius $R_o = 20$ cm and inner radius $R_i = 18$ cm. A plastic strip initially straight is inserted into the channel to simulate the planar deformation of the beam. Referring to Fig. 2, the boundary conditions are that the left end (A) is clamped and that the right end (B) is pinned to a cube made of paper free to move along the channel, and thus a compressive load is applied at B and subsequent buckling shapes are observed. As the load is increased, the corresponding progressive stable configurations of the strip (in equilibrium state) are shown from Fig. 2(a)–(c). These qualitative observations are compared to our simulation results (see Fig. 5).

3. Simulation

Traditional methods used to solve the buckling problem of beams relies on the solution of the differential equation or on finding the minimum potential energy (Budynas, 1998). For the latter methods, the resulting minimization problem is very challenging to solve when the beam is geometrically constrained in a curved channel. In this work, we will solve the problem by employing a new technique based on a Dynamic Programming Approach (Bellman, 1954) called the Viterbi algorithm (Forney, 1973). While this algorithm is well known in the information theory literatures. The fact that it can be used for the solution of constrained beam buckling problems was already noted in previous work. Doraiswamy et al. (2012) and Narayanan and Srinivasa (2009). For the cantilever beam problem, the differential equation together with the initial conditions has a special Markov structure that can be gainfully exploited. The Markov structure makes the dynamic programming approach suitable for this problem and allow us to sequentially optimize it.

To summarize, the key idea of the Viterbi algorithm is to efficiently search the set of possible configurations of the beam and locate the configuration with minimum potential energy. The major advantage of this technique is that, since it is based on a global search and not on a calculus based approach, it is guaranteed to find the most probable configuration among all the admissible possibilities. However, for non-linear elastic problems, there may exist many different equilibrium states for the system as long as the potential energy reaches a local minima (Budynas, 1998). Although we are finding the lowest energy state among many possible configurations, we should observe that the Viterbi algorithm can be extended to find all local minima by employing a technique



(a) Pattern 1 (b) Pattern 2 (c) Pattern 3

Fig. 2. Experimental observations of deformation of an elastic strip in a quarter circular channel. The left end of the beam is clamped and the right is pinned to a small piece of paper compressed into a cube. Note that the progressive appearance of buckled patterns are compatible with simulations in Fig. 5.