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Extended finite element method for strong discontinuity analysis of strain localization of non-associative plasticity materials

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ABSTRACT

The purpose of this paper is to propose a new development of the extended finite element method (XFEM) for carrying out the strong discontinuity analysis of strain localization of non-associative plasticity materials. The displacement is assumed to possess a Heaviside jump at the localization interface and the strain becomes bounded measure including a Dirac-delta function, but the traction must be continuous at the localization interface. First, the explicit expressions for the initiation condition and the slip direction and motion direction for the localization interface are derived from discontinuous bifurcation analysis. Second, the linearly and exponentially decreasing cohesive models are proposed respectively as the product of the combination of the continuum plasticity model and strong discontinuity kinematics, which relates the cohesive softening properties to the cohesive surface energy (or called cohesive fracture energy). Third, a linear cohesive/friction coupled model is proposed for the frictional contact interface based on the Mohr–Coulomb frictional law. The evolution of zero-thickness localization band (or called slip line) is shown to be equivalent to the crack initiation and propagation. Fourth, the finite element formulation is derived based on the XFEM to capture the displacement discontinuity. It is shown that numerical analysis for predicting the evolution of the localization interface ultimately falls into the framework of the XFEM with the embedded zero-thickness cohesive model or cohesive/friction coupled model. The cohesive segment method in the XFEM is used to simulate the initiation and propagation of the localization interface along the slip direction across the enriched elements. Numerical convergence due to cohesive/friction softening behaviors is solved by viscous regularization. Finally, numerical results on two typical cases including the single notched four-point bending model and the soil slope stability model under compression demonstrate the proposed methodology by studying the effects of the mesh size and cohesive surface energy on the evolutive localization interface and the load–displacement responses.

1. Introduction

Strain localization as a real physical phenomenon appears widely in the rock, soil and metal such as the shear band, slip line and rupture zone, which are often followed by the remarkable reduction of the overall strength of solid (Hill, 1962; Thomas, 1961; Palmer and Rice, 1973; Rudnicki and Rice, 1975; Höeg and Prévost, 1975; Prévost, 1984). Localization means high-gradient deformation fields appear at a very narrow zone, which is often associated with strain softening behavior and material instability represented by the ill-posedness of the stress governing equation. For example, the width of the localization band (or called shear band) for the granular soil is only about the order of ten grain diameters. This means the finite-width nature of the localization band creates an inherent length scale (Loret et al., 1995; Wolf et al., 2003; Sanborn and Prévost, 2011). Across the localization band, there exists a discontinuity in the displacement gradient (or velocity gradient) field, which is usually referred as *the weak discontinuity*. However, the classic rate-independent plasticity theory and finite element analysis (FEA) based on the continuum mechanics do not provide an objective description of the strain localization due to the absence of internal length scale. In fact, when the localization appears, the plastic flow is localized to those elements that experience strain softening in the FEA, and most of the energy dissipation also takes place over there. When the mesh continues to be refined, zero characteristic length is approached.

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This fails to transmit fully the deformation information from the localized elements to the entire continuum body, leading to mesh sensitivity problem.

This problem was typically circumvented by using some theoretical methods which introduce a length scale into the stress governing equation, including the nonlocal integral theory (Eringen, 1981; Pijaudier-Cabot and Bažant, 1987), the nonlocal gradient theory (Triantafyllidis and Aifantis, 1986; Sluys et al., 1993; Iacono et al., 2008) and the viscoplastic models (Needleman, 1988; Loret and Prévost, 1990; Prévost and Loret, 1990). Unfortunately, even though the length scale is included in the governing equation, the mesh still has to be fine enough. Besides, the actual width of the localization band was found by Prévost and Loret (1990) to be related to the length scale. However, accurate calibration of the length scale requires some special experimental approaches (Abu Al-Rub and Voyiadjis, 2004). In addition, adaptive mesh techniques (Ortiz and Quigley, 1991; Deb et al., 1996) were adopted to capture the evolution of the localization band by refining the mesh sizes in the regions of localization, which yet show expensive modeling and computational cost.

Perhaps a more favorable strategy would be to treat the width of the localization band to be zero as a limiting case since the width of band is generally much smaller than the sizes of solids. In this case, the kinematics of the *weak discontinuity* reduces to that of the *strong discontinuity* (Simo et al., 1993). The weak discontinuity contains continuous displacement but discontinuous strain (or acceleration). By comparison, the strong discontinuity contains discontinuous displacement (or velocity). From the macroscopic scale, damage and failure of solids gradually translate to the displacement discontinuity (Oliver et al., 2014). The concept of strong discontinuity is introduced that ensures the classic plasticity theory can be compatible with the discontinuous displacement field arising from the localization interface, which avoids the need to introduce the length scale and to refine the mesh sizes in the region of localization because the energy dissipation is calculated over measure zero at the localization interface.

In general, the strong discontinuity analysis which is now classified into the field of computational failure mechanics (Oliver et al., 2014) includes three important issues: 1 the initiation condition and the slip and motion directions of the localization band, 2 the constitutive model that governs the progressive failure of the post-localization interface and 3 the finite element formulation that solves the standard and enriched node displacements. For the first issue, Simo et al. (1993), Armero and Garikipati (1996), Oliver (1996a), Oliver et al. (1999), Oliver (2000), Oliver and Huespe (2004), Oliver et al. (2006), Sánchez et al. (2006), Borja et al. (2000), Borja and Regueiro (2001), Borja (2002), Regueiro and Borja (1999, 2001), Wells and Sluys (2001), Foster et al. (2007), Linder and Armero (2007) and Sanborn and Prévost (2011) carried out the strong discontinuity analysis for associative or non-associative plasticity materials under small deformation or finite deformation. It had been concluded by Simo et al. (1993), Oliver (1996a), Oliver et al. (1999), Regueiro and Borja (2001), Borja (2002), Armero and Garikipati (1996), Linder and Armero (2007) and Sanborn and Prévost (2011) that the initiation condition of strain localization by the strong discontinuity analysis is that the determinant of the elastic perfectly plastic acoustic tensor is zero. In addition, Leroy and Ortiz (1989) and Oliver (1996b) proposed tracking algorithms to search the slip and motion directions of the localization interface by minimizing the determinant of the acoustic tensor for associative plasticity materials. Wells and Sluys (2001) and Sanborn and Prévost (2011) further extended these tracking algorithms to non-associative plasticity materials by FEA. By comparison, Armero and Garikipati (1996), Oliver et al. (1999) and Regueiro and Borja (1999, 2001) derived the explicit expressions for the initiation condition and the slip and motion directions based on the traction continuity requirement at the localization interface. For the second issue, Simo et al. (1993), Armero and Garikipati (1996), Oliver (1996a), Oliver et al. (1999), Oliver (2000), Borja and Regueiro (2001), Regueiro and Borja (2001) and Borja (2002) derived discrete post-localization constitutive models from the continuum plasticity model, which were related to the cohesive surface energy (or called cohesive fracture energy). Oliver (2000) further showed these discrete models can be regarded as a projection of the continuum plasticity model on the discontinuous interface. By comparison, Sanborn and Prévost (2011) introduced directly the Mohr–Coulomb frictional contact law as the post-localization interface model which was also related to the surface energy. From the theoretical perspective, there should be a continuous transition for the interface from the cohesive state to the frictional contact state. Foster et al. (2007) showed there are two phases for the localization interface of geomaterials. The first phase is the degradation of cohesive strength as the coherent macrocrack form which was called as "slip weakening" and the second phase is the evolutive frictional response along the interface during the slip weakening. However, it is actually hard to strictly distinguish the cohesive or frictional phase from the physical perspective. Thus, a favorable approach would be to couple the cohesive model with the friction to explain the evolution of the localization interface. However, the explicit cohesive/friction coupled model is little established in the strong discontinuity analysis. For the third issue, numerical implementation for tracking the strong discontinuity path requires the use of robust finite element formulations. Generally, two methods have been used to model discontinuous fields in the FEA: the assumed enhanced strain method (AES) and the extended finite element method (XFEM). Based on the three-field Hu-Whashizu variational principle, Simo et al. (1993), Armero and Garikipati (1996), Oliver (1996a), Oliver et al. (1999) and Armero and Kim (2012) developed the AES to model the discontinuity, in which the enrichment was done on the element level with resulting extra unknowns statically condensed out of the equations. The AES falls into the framework of the Petrov-Galerkin finite element formulation with embedded discontinuity. By comparison, in the XFEM proposed by Moës et al. (1999) and Dolbow et al. (2000), the concept of partition of unity was used and the enrichment was done on the global level by adding extra unknowns to the global set of equations. The XFEM is classified into the Bubnov-Galerkin formulation. Borja (2008) made a comprehensive comparison between these two methods. Compared with the XFEM, the AES requires the crack must pass across an entire element and there is also no method to locate the crack tip inside elements. For the strong discontinuity analysis including the crack initiation and propagation, the XFEM shows a large advantage over the AES.

This research concentrates on theoretical modeling and numerical analysis on the strain localization issue for non-associative plasticity materials by strong discontinuity approach. The main purpose is to propose a new development of the XFEM in simulating the initiation and propagation of the localization band or crack. The theoretical work is to derive the initiation condition and slip direction of the localization interface and to propose linear and exponential cohesive interface models and a linear cohesive/friction coupled model. These two kinds of cohesive models are the first new contribution of this paper. Then, the second new contribution of this paper is to advance the strong discontinuity research into the framework of the XFEM with embedded cohesive interface models. The remaining of this paper is organized as follows. Section 2 briefly summarizes the standard non-associative plasticity theory. Section 3 derives the explicit expressions for the initiation condition and the slip and motion directions of the localization interface, and proposes the linearly and exponentially decreasing cohesive models respectively, and a linear