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A new marching ridges algorithm for crack path tracking in regularized media

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ABSTRACT

Tracking algorithms are used to predict crack paths in structures modeled with the finite element method, in such a way that the paths do not depend on the selected mesh. For regularized media, the simplest methods rely on scalar variables, somehow related to material degradation. Despite their simplicity, they suffer from a major limitation: they allow the crack to initiate and propagate in only one direction. Consequently, such approaches usually fail in case of crack branching or crack initiation inside the structure. To overcome this difficulty, we propose a new crack path tracking algorithm. It is designed to simultaneously detect several local maxima of a degradation-related variable by following the associated ridge lines. That is why the algorithm proposed in this paper could be referred to as a *marching ridges algorithm*. The performance of the proposed approach is illustrated by three numerical examples within different frameworks. The first ones show that the algorithm can be used to insert crack increments during a ductile failure computation with a quasi-static implicit resolution procedure, in 2D and 3D. The last example proves that the algorithm can be used as a post-processing tool to capture dynamic crack branching from a damage distribution image only.

1. Introduction

Within the framework of continuum damage mechanics, the failure of the underlying microstructure is usually described in an average sense by a continuous model through a degradation-related variable. This continuous description is acceptable up to the onset of fracture. But to model crack propagation, a continuous model is no longer sufficient since it cannot properly describe the kinematics associated with a crack opening (and possibly closing again). Representing this discontinuity (whichever technology is chosen) requires first and foremost to be able to precisely determine the location of the discontinuity interface. This can be achieved through the so-called *crack path tracking algorithms*.

1.1. Tracking algorithms in the literature

Several tracking algorithms are available in the literature. According to Oliver et al. (2004), they can be classified into three groups.

- Local (or propagation) algorithms are based on the geometrical propagation of the crack from an initial crack front, by using local information about the propagation direction.
- Global tracking algorithms trace all possible crack paths at once, on the basis of the information provided by a global propagation field. For example, Oliver et al. (2004) proposed a global algorithm based on solving an additional heat conduction-like problem for tracking multiple crack lines/surfaces in geomaterials. This global approach has also been followed by Huespe et al. (2009) for small deformations and in Huespe et al. (2012) for finite strains.
- Tracking algorithms based on the level set concept (Stolarska et al., 2001; Moës et al., 2011) share some of the characteristics of local and global algorithms.

Local algorithms are most commonly used in the literature. This may be explained by the fact that these algorithms are generally cost effective, whereas propagating level-set methods and global tracking algorithms can be "cumbersome and code-invasive", according to Oliver et al. (2014).

Different local algorithms have been proposed to determine crack orientation for strain-localization problems. Their specificities are associated to the type of formulation chosen.

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With a standard formulation (i.e. a local one), the uniqueness of the solution may be lost at some point (which causes spurious mesh dependence, i.e. the orientation of the localization band is dependent on the orientation and the size of the mesh). However, this dependence can be overcome by i/ determining the moment when the solution uniqueness is lost, ii/ determining the orientation of the discontinuity interface, and iii/ immediately enriching the elements which should be crossed by the discontinuity interface or introducing cohesive elements. For example, Areias and Belytschko (2005) suggest to study, at the end of each converged load increment, the incremental stability and uniqueness of the solution. When needed, the orientation of the discontinuity interface is then determined thanks to the spectral decomposition of an average strain tensor and a cohesive element is inserted. Oliver et al. propose to use a bifurcation analysis to determine the moments corresponding to a loss of ellipticity of the equations. Then the orientation of the discontinuity interface is obtained by algorithms which usually give two admissible local directions (Oliver and Huespe, 2004; Oliver et al., 2010). In order to select the best direction, a local crack path tracking algorithm based on a *crack-path-field technique* has recently been proposed in combination with a *strain-injection technique* (Oliver et al., 2014). It requires the resolution of a secondary problem which involves the smoothing of both the strongly localized scalar field and its directional derivative.

Another way to solve the mesh dependence issue is to resort to regularized formulations (e.g. nonlocal models, gradient-regularized models or Cosserat models). For these regularized formulations, the width of the localization band is controlled by an additional characteristic length introduced in the continuum constitutive model. The band then encompasses several elements. Within that band, strains and degradation-related variables reach a maximum value, so phenomenologically based criteria can be used to determine the crack orientation. As with this formulations the uniqueness of the solution is never lost, these criteria can be used later on during computation, i.e. when material degradation is more important. The most common technique consists in using a weighted average of one of the variable associated with material degradation. For example, Wells and Sluys (2002) assumed that the discontinuity would extend from its tip in the direction in which the effective stress is maximum for the Perzyna viscoplastic model. Simone et al. (2003) relied on the direction of maximum accumulation of the nonlocal equivalent strain in a V-shaped window ahead of the crack tip for 2D crack propagation. Mediavilla et al. computed the average direction of maximum damage over several radii centered at the crack tip for a 2D implicit gradient-enhanced nonlocal model (Mediavilla, 2005; Mediavilla et al., 2006). A similar criterion has been used by Broumand and Khoei (2013). An extension of Mediavilla's work has been carried out by Javani for 3D crack propagation (Javani, 2011). However, these criteria based on a weighted average seem to be limited to cases where a crack can only propagate in a single direction from the considered crack front (in particular, to the authors' knowledge, they have not been applied in case of crack branching). Their use would thus be mainly restricted to cases where a crack initiates on the external boundary and propagates inside the structure.

1.2. Motivation and objectives of the work

In this paper we describe a crack path tracking algorithm that has been recently proposed (Feld-Payet, 2010; Feld-Payet et al., 2012). Its originality lies in the study of the sign of a particular scalar product to follow ridges. The scalar product involves the gradient of a regularized degradation-related field and an orthoradial vector defined on a polar grid centered on the crack tip. To the authors' knowledge, this type of algorithm is completely new within continuum mechanics. It is however more common in the computer vision field where it is qualified as a marching ridges algorithm.

Within continuum mechanics, this tracking algorithm still bears some similarities with the crack path field technique proposed by Oliver et al. (2014). For both methods the crack is viewed as the zero level set of a directional derivative. However, the major difference in the described algorithm resides in the discretization of the considered plane with a polar grid. It allows both to immediately define the direction on which the gradient should be projected and to avoid the resolution of an additional sub-problem. The proposed algorithm also presents the advantage to deal with a scalar field, instead of a tensor, as in Areias and Belytschko (2005). Compared to the other criteria studying a scalar field for regularized media, the most important difference is the consideration of the *gradient* of the degradation-related field whereas, to the authors' knowledge, the other algorithms rely on the *scalar field* itself.

These characteristic features allow the proposed criterion to *simultaneously capture several directions* while being versatile regarding both the constitutive behavior and the software. Indeed, it can be applied to any regularized constitutive behavior since it can deal with any scalar field related to material degradation. Such an algorithm can be applied as a post-processing tool on any kind of mechanical field map. Consequently, it is highly generic and its implementation within any commercial FE software is straightforward.

The objective of this work is the presentation of a new crack path tracking algorithm which aims at matching the crack path with the direction of maximum material degradation, regardless of the constitutive behavior, of the regularization method or their ability to reproduce the experimental results. To introduce this new approach as briefly and simply as possible (i) only mode I-II fracture is considered and (ii) numerical examples involving a finite element computation will use a single constitutive behavior with an implicit gradient enhanced regularization within the small strain framework.

The focus of this work is:

- first to explain the theoretical bases of the algorithm (Section 2.1) and its application to crack propagation in 2D (Section 2.2) and in 3D (Section 2.3);
- then to illustrate its numerical performance both during a finite element computation and as a post-processing tool. After detailing briefly the framework of the presented results (Section 3.1), a 2D test case proves the ability of the algorithm to match the crack path with the ridge of a damage function, even for a non-straight ridge (Section 3.2). Then a 3D test case illustrates (for a slightly different damage-related function) the ability of the algorithm to deal with crack initiation at a specimen's center (Section 3.3). Finally Section 3.4 proves that the proposed algorithm can capture crack branching thanks to the analysis of an image representing a nonlocal damage distribution after dynamic loading.

More details on the algorithm, the chosen constitutive behavior and some aspects of the mesh adaptivity strategy can be found in the appendices.

2. A new marching algorithm for ridge detection

2.1. Theory

2.1.1. Matching crack paths and ridges

Consider, for a two-dimensional problem, a field associated to material degradation (e.g. a damage field, or a field associated to void growth,...), denoted f. From a mechanical point of view, the