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Steady shock waves in porous plastic solids

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ABSTRACT

The affect of material porosity on propagation of shock waves in solids is examined in the context of finite strain, associated plasticity, with porosity incorporated via the Gurson model and accounting for material hardening. Setting is analogous to the fluid dynamics piston shock model so that deformation of the semi-infinite medium is permitted only in the longitudinal direction. The steady response, which develops by imposing constant piston velocity in either tension or compression, is examined by sectors mapping of the characteristic velocity as determined by the constitutive model. It is shown that even the slightest levels of initial porosity can have an appreciable effect on field response, inducing destructive unsteady behavior accompanied by increased shock dissipation. Numerical illustration of limit velocities at appearance of a plastic shock and at onset of that unsteady behavior are presented, showing that material porosity delays initiation of plastic shock waves and promotes higher energy consumption which may, in turn, enhance protective capabilities.

1. Introduction

The role of material porosity in propagation of shock waves under extreme loading conditions is investigated. A longitudinal deformation field, analogous to the piston shock field in fluid dynamics, is considered to facilitate investigation of steady shock wave propagation in porous solids. The present work builds on a previous study (Cohen and Durban, 2014) which has shown that the specific geometry of longitudinal deformation has a profound effect on the dynamic response. Considering the three-dimensional geometry of the piston shock field, the analysis therein shows that the dynamic response may consist of both an elastic precursor and a plastic shock separated by a continuous elastoplastic range. That behavior has been observed experimentally (Marsh, 1980; Longy and Cagnoux, 1989; Davison and Graham, 1979), however it is not fully captured by the more frequently considered 1*D* and 2*D* geometries. Note that though the geometric setting considered here is 3*D*, the deformation field propagates only along the longitudinal direction, as will be detailed in the next section.

Though available experimental and theoretical research on shock processes in solids (Marsh, 1980) is sufficient to delineate the fundamental behavior, some questions remain open. In the context of protective structures, for example, it is not clear whether shock waves are essentially destructive and should be avoided to maintain structural integrity or, alternatively, have a decelerating effect on the penetrator due to increased energy absorption by dissipation, as the wave drag effect in fluid dynamics. Which of these two contradicting phenomena dominates the response depends on the penetration velocity and the material properties. Incorporation of measured levels of material porosity can influence the response substantially and facilitate design of more efficient protective structures in the future.

The present study attempts at a benchmark problem to determine the role of porosity in the steady dynamic material response by evaluation of dissipation effects and critical velocities for appearance of a plastic shock wave. Therefore the analysis in Cohen and Durban (2014) is extended to include material porosity modeled by the generalized (Gurson, 1977) model. As a first step, the longitudinal stress-stretch response of the porous solid, constrained in the transverse direction, is examined. Next, evaluation of characteristic velocity and sectors mapping of the discontinuous and continuous zones is conducted, leading to the complete field response by applying jump conditions and continuous self-similar solutions, respectively. Energy considerations then result in a measure for the shock dissipation of the porous field, at different levels of initial porosity, as compared with that of the nonporous field.

A pioneering study on propagation of longitudinal, small strain, dynamic deformation in elastoplastic solids dates back

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to von Karman and Duwez (1950). The longitudinal pattern considered therein was of a tensioned bar allowing changes in the cross sectional area and with no transverse stress. A more recent study, by Knowles (2002), considered the finite strain tension response for a similar deformation pattern but with a rubber-like material. In that study conditions for appearance of shock waves and jump relations across the discontinuity were formulated.

In the longitudinal deformation pattern considered here the solid is constrained so that deformation in the transverse direction is not permitted, implying that the transverse stresses are active. Therefore, motion is purely uniaxial in a spatial setting. Despite the difference in geometry, the formulation of the longitudinal equation of motion and compatibility relations in the longitudinal deformation patterns is similar to that in Knowles (2002), implying an identical governing equation, obtained in the form of a nonlinear longitudinal wave equation for arbitrary constitutive relations. Thus resulting in conditions for appearance of shock waves and the associated continuous and discontinuous solutions (von Karman and Duwez, 1950; Knowles, 2002). Since the constitutive response depends on the deformation pattern, fundamental differences between the present field response and that in a different longitudinal deformation pattern may appear as shown by a comparison in Cohen and Durban (2014).

Fomin and Kiselev (1997) investigated the appearance of shock waves in porous solids by numerical simulation of plate impact tests. Therein, the porosity was incorporated via a modified Gurson model. Appearance of shock waves in porous media is discussed in Davison and Graham (1979) for materials of very low density. Drumheller (1998) considered the dynamic response of saturated porous media. Studies on longitudinal shock waves in nonporous media are reviewed in Cohen and Durban (2014) and in Howell et al. (2012). It should be noted that the constitutive response considered here applies for ductile materials with low levels of initial porosity, as for example metals with pores as imperfections and sintered metals.

The Gurson (1977) model employs averaging methods to account for the effect of material porosity via a continuum approach assuming homogenous distribution of void volume fraction. Hence, the present constitutive model is not sufficient for investigation of the response of granular media, materials with high levels of the void volume fraction or rate-sensitive materials. Additionally the present approach is purely mechanical employing standard principals of the theory of plasticity and does not account for thermo-mechanical coupling with the material response determined completely by the Gurson (1977) constitutive equation. Therefore it does not employ an equation of state to analyze the dynamic response, as does a substantial body of literature in the field (Addessio and Johnson, 1993; McQueen et al., 1970; Brown et al., 2007; Herrmann, 1969).

2. Problem formulation

An instructive pattern that reveals the fundamental nature of shock processes in porous media is the steady-state longitudinal deformation field. The face of a semi-infinite body is put to motion at constant velocity and, while strain is permitted only in the longitudinal direction, transverse stresses preserve uniformity in the cross-sectional area. Constraints are analogous to those of the elementary shock tube in gas dynamics, discussed in Cohen and Durban (2014) and illustrated on Fig. 1. The enforced end velocity (V) is associated with constant applied (dimensionless) stress (σ) which may be either tension ($V < 0$) or compression ($V > 0$). Throughout the formulation stresses are nondimensional with respect to elastic modulus (E). Briefly recapitulating the formulation in Cohen and Durban (2014), we combine the compatibility equation with the equation of motion, to find that the deformation field is governed by the nonlinear wave equation

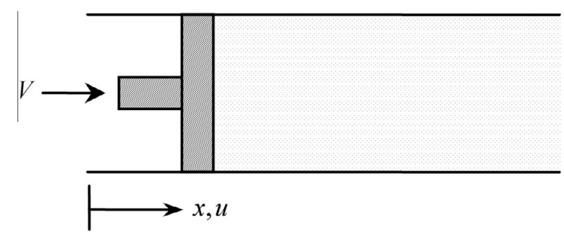

Fig. 1. The shock tube field with compressed porous media is schematically illustrated, with the Lagrangian longitudinal coordinate (x), the longitudinal displacement (u) and the enforced piston velocity (V).

$$u_{,tt} = C^2 u_{,xx} \tag{1}$$

where x is the Lagrangian coordinate, u is the longitudinal displacement, t denotes time and C is the characteristic wave velocity. Considering constitutive laws where σ depends only on the stretch $a = 1 + u_{,x}$, namely $\sigma = \sigma(a)$, the characteristic velocity is a function of the stretch, obeying the relation

$$C^2 = C_E^2 \sigma' \tag{2}$$

where the superposed prime represents differentiation with respect to stretch (a) and $C_E = \sqrt{E/\rho_o}$ is the elastic wave speed in a long rod, with ρ_o denoting the initial macroscopic density. We limit the discussion to stable material response with positive stress-stretch slope ($\sigma' > 0$), thus preserving the hyperbolic nature of the equation.

Steady solutions of the nonlinear wave Eq. (1), if exist, may be either continuous or purely discontinuous with the possibility of combination of both, depending on variation of characteristic velocity with applied load, as reflected by the stress-stretch curve slope according to (2). Conditions for appearances of discontinuity are obtained by evaluation of the characteristic slopes (Courant and Friedrichs, 1948)

$$\begin{aligned} &\text{in compression}: \ \sigma'' < 0 \\ &\text{in tension}: \ \sigma'' > 0 \end{aligned} \tag{3}$$

implying convexity/concavity of the stress-stretch curve in compression/tension, respectively. A more recent derivation of the shock condition, in a tensioned longitudinal bar can be found in Knowles (2002). In that study changes in the cross-sectional area of the bar are permitted while the bar is free of transverse load. However, since motion is considered only in the longitudinal direction, formulation of (1)–(3) is analogous to that in Knowles (2002) with differences imposed by the stress-stretch relation $\sigma = \sigma(a)$, which in the present analysis is derived assuming a three-dimensional geometry.

If shock condition (3) holds and the field is purely discontinuous, then segments with constant stretch and velocity are divided by discontinuity. Thus, requiring conservation of mass and momentum across the shock we write the jump conditions

$$[\![u]\!] = 0 \tag{4}$$

$$s[\![v]\!] + C_E^2[\![\sigma]\!] = 0 \tag{5}$$

also known as the Hugoniot conditions, where s is the Lagrangian shock wave velocity and $v = u_{,t}$ is the material velocity. Note that the double bracket notation implies the jump of the inserted quantity across the discontinuity.