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Interphase zone effect on the spherically symmetric elastic response of a composite material reinforced by spherical inclusions

Roberta Sburlati [a,*], Roberto Cianci [b]

[a] Department of Civil, Chemical and Environmental Engineering, University of Genova, Via Montallegro 1, 16145 Genova, Italy
[b] DIME – Sez. Metodi e Modelli Matematici, University of Genova, Piazzale J.F. Kennedy, pad.D, 16129 Genova, Italy

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ABSTRACT

This work deals with the problem of modeling the effect due to an interphase zone between inclusion and matrix in particulate composites to better estimate the bulk modulus of materials with inclusions. To this end, in this paper the problem of a body containing a hollow or solid spherical inclusion subjected to a spherically symmetric loading is investigated in the framework of the elasticity theory. The interphase zone around the inclusion is modeled by considering the elastic properties varying with the radius moving away from the interface with inclusion and, asymptotically approaching the value of the homogeneous matrix. The explicit solutions are obtained in closed form by using hypergeometric functions and numerical investigations are performed to highlight the localized effects of the graded interphase in the stress transfer between inclusion and matrix. Finally, the exact solutions are used to estimate the effective bulk modulus of a material containing a dispersion of hollow or solid spherical inclusions with graded interphase zone.

1. Introduction

In many particulate-reinforced composites the interface between the matrix and the inclusion plays an important role to estimate their effective elastic properties. From the first papers in which inclusion/matrix interface are assumed perfectly bonded together, a number of researchers have attempted to account for interface effects by using analytical and numerical approaches. Furthermore, in some cases, the inclusions are surrounded by thin interface layers whose thickness is usually much smaller than the inclusion sizes and, as a consequence, the properties of the interface layer do not significantly affect the elastic constant of the composite; on the contrary, if the interface thickness is comparable with the inclusion size, the effect of the interface zone properties may be substantial on the evaluation of the elastic properties of the composite: i.e. nanoparticle reinforced materials (Sevostianov and Kachanov, 2007), hollow particle filled composites (Tagliavia et al., 2011), as well as concrete (Lutz and Zimmerman, 1996a).

In 1964 Hashin and Rosen developed a model to take into account the interphase effects, by considering an homogenous interphase zone around the inclusion with different elastic properties from those of matrix or inclusion; but, starting from 1990, some authors presented models in which a distinct inhomogeneous interphase zone with step by step (Herve and Zaoui, 1993) or smooth variation of the elastic moduli is introduced (Jayaraman and Reifsnider, 1992). Indeed, Lutz and Zimmerman (1996a) investigated the transfer of the stress between solid spherical inclusion and matrix to predict the bulk modulus of the composite, modeling the interphase as a layer with elastic properties variable in the radial direction but with a smooth transition between interphase and matrix. A similar approach is also used by authors to estimate the thermal/electrical conductivity of particulate composites that contain a dispersion of solid spherical inclusions (Lutz and Zimmerman, 1996b, 2005).

Though great of attention has been paid to the study of solid inclusions with interphase zone (Wang and Jasiuk, 1998; Shen and Li, 2003, 2005); starting from 1970, the demand for light weight and high strength materials have increased the use of micro hollow spheres (Lee and Westmann, 1970; Huang and Gibson, 1993). Some papers are devoted to estimating the elastic properties of hollow-sphere-reinforced composite with perfect interface (Bardella and Genna, 2001; Marur, 2005; Porfiri and Gupta, 2009) or imperfect interface (Tagliavia et al., 2011; Marur, 2014). But, experimental results and application to nanocomposites also suggest investigating the effects of an inhomogeneous interphase zone around inclusion which, when the inclusion geometry is

* Corresponding author.
E-mail addresses: roberta.sburlati@unige.it (R. Sburlati), roberto.cianci@unige.it (R. Cianci).

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comparable with the thickness of the interphase, allows to a stress concentration around the inclusion that may affect the elastic properties of the composite (Shen and Li, 2005; Sevostianov and Kachanov, 2007). As far as the author knows, in the literature are not present elastic analytical solutions in closed form for spherical solid or hollow inclusions taking into account graded interphase effects.

This work deals with the problem introduced above. In particular, in this paper an elastic analytical solution is developed in closed form for the problem of hydrostatic pressure of a homogeneous body containing a hollow spherical inclusion, and the solution for solid inclusion is determined as a consequence. In Section 2, assuming spherically symmetric loading, the mathematical model is formulated with the introduction of an inhomogeneous interphase around the inclusion to describe the transition zone between the inclusion and the matrix; in other words, this interphase is considered as a functionally graded material (FGM) with radial variation of the elastic moduli from the center of the inclusion that asymptotically assumes the homogeneous elastic properties of the matrix. Similar elastic solutions are obtained by the author to study both the effects of a FGM thin layer on the inner surface of a cylinder under pressure (Sburlati, 2012) and how the stress concentration factor is altered by a layer around a hole in a plate subjected to uniaxial load (Sburlati, 2013). In Section 3 the analytical solution is obtained in the framework of the elasticity theory by applying the theory of hypergeometric functions (Abramovitz and Stegun, 1965; Erdelyi et al., 1953). The explicit solution is written in closed form in Section 4 and permit us to understand the manner in which the interphase affects the stress transfer between spherical inclusions and matrix. The solution is also written in detail for solid inclusion without interphase. In Section 5 the in closed-form analytical solution is used to obtain effective bulk modulus of the equivalent composite, by adopting the energy approach (Willis, 1981). Numerical examples are performed in Section 6 to highlight the effects of the geometric and physical properties of the interphase on the stress transfer and the bulk modulus.

The results obtained in the paper may be the starting point to predict other elastic properties of the equivalent homogeneous composite as the thermal expansion coefficient and the conductivity. Furthermore, we remark that the interphase region may describe phenomena occurring during the material processing stage but the understanding of the role of the interphase may also permit us to tailor the interphase properties around the inclusions in order to enhance the mechanical properties of composites (i.e: Paskaramoorthy et al., 2009; Batra, 2011; Yao et al., 2013; Zhang et al., 2013).

2. Mathematical model

In this section a two-phase model is introduced to evaluate the effects of a graded interphase on the stress transfer mechanism between a spherical inclusion and a matrix in a particulate composite subjected to a spherically symmetric loading. We start to analyze the case of hollow spherical inclusion and then we obtain the solution for solid spherical inclusion.

The three-dimensional problem studied for the hollow spherical inclusion case is shown in Fig. 1. A single, homogeneous, isotropic, spherical hollow inclusion of inner radius a and outer radius b is embedded in a matrix and subjected to a remote hydrostatic load p. The matrix is modeled as an isotropic graded material in radial direction in order to describe an interphase zone of thickness t around the inclusion in which the elastic properties, moving away from the interface with the inclusion, asymptotically approach the constant values of the elastic properties of the matrix without

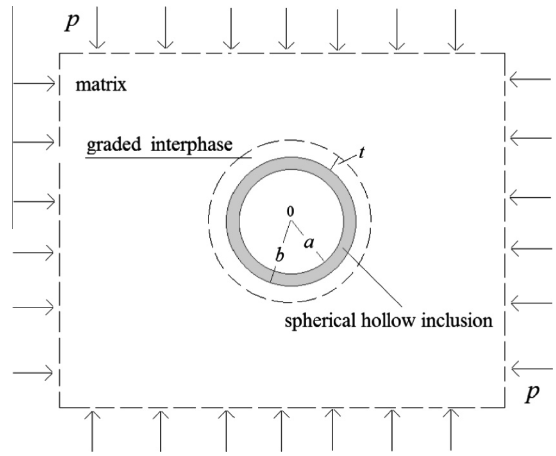

Fig. 1. Sketch of the mathematical problem studied.

inclusion. The interface between inclusion and interphase is assumed still distinct since the inhomogeneous region is restricted to the matrix region around the inclusion. Again, the graded interphase zone and the matrix are described with a power variation law in radial direction.

By using a spherical coordinate system $(0; r, \theta, \phi)$, the load symmetry permits us to reduce the problem to determine the radial displacement u and the radial and hoop stresses: $\sigma_r, \sigma_\theta = \sigma_\phi$. The stresses and the displacement in inclusion and in matrix will be index with the superscript (i) and (m) respectively. Then, the power laws for the Lamé moduli in the isotropic graded interphase/matrix region (m) are assumed as

$$\lambda(r) = \lambda_m + \bar{\lambda}\left(\frac{b}{r}\right)^{\beta}, \quad \mu(r) = \mu_m + \bar{\mu}\left(\frac{b}{r}\right)^{\beta}, \tag{2.1}$$

where

$$\bar{\lambda} = \lambda_{ip} - \lambda_m, \quad \bar{\mu} = \mu_{ip} - \mu_m. \tag{2.2}$$

and the Lamé constants λ_m, μ_m are the asymptotic values of the matrix and λ_{ip}, μ_{ip} the values of the interphase elastic constants at the interface between inclusion and interphase $(r = b)$. The parameter $\beta > 0$ is called the inhomogeneity parameter and permits us to control the interphase thickness, indeed high beta values correspond to small interphase thickness and viceversa. Furthermore, the sign of the quantities $\bar{\lambda}$ and $\bar{\mu}$ describe *hard* or *soft* interphases in which the elastic properties respectively decrease $(\bar{\lambda} > 0, \bar{\mu} > 0)$ or increase $(\bar{\lambda} < 0, \bar{\mu} < 0)$ away from inclusion/interphase interface.

To complete the mathematical problem, we write the boundary and the asymptotic conditions in the following form (Love, 1944):

$$\sigma_r^{(i)}(a) = 0, \tag{2.3}$$

$$\lim_{r\to\infty} \sigma_r^{(m)}(r) = -p. \tag{2.4}$$

where $p > 0$ is the remote hydrostatic pressure. Then, the interphase and the inclusion in $r = b$ are assumed perfectly bonded together as

$$\sigma_r^{(i)}(b) = \sigma_r^{(m)}(b), \quad u^{(i)}(b) = u^{(m)}(b). \tag{2.5}$$

3. Analytical solution

The field elasticity equation in terms of radial displacement, for inhomogeneous isotropic material with elastic properties