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On the elastic-wave imaging and characterization of fractures with specific stiffness

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ABSTRACT

The concept of topological sensitivity (TS) is extended to enable simultaneous 3D reconstruction of fractures with unknown boundary condition and characterization of their interface by way of elastic waves. Interactions between the two surfaces of a fracture, due to e.g. presence of asperities, fluid, or proppant, are described via the Schoenberg's linear slip model. The proposed TS sensing platform is formulated in the frequency domain, and entails point-wise interrogation of the subsurface volume by infinitesimal fissures endowed with interfacial stiffness. For completeness, the featured elastic polarization tensor – central to the TS formula – is mathematically described in terms of the shear and normal specific stiffness (κ_s, κ_n) of a vanishing fracture. Simulations demonstrate that, irrespective of the contact condition between the faces of a hidden fracture, the TS (used as a waveform imaging tool) is capable of reconstructing its geometry and identifying the normal vector to the fracture surface without iterations. On the basis of such geometrical information, it is further shown via asymptotic analysis – assuming "low frequency" elastic-wave illumination, that by certain choices of (κ_s, κ_n) characterizing the trial (infinitesimal) fracture, the ratio between the shear and normal specific stiffness along the surface of a nearly-planar (finite) fracture can be qualitatively identified. This, in turn, provides a valuable insight into the interfacial condition of a fracture at virtually no surcharge – beyond the computational effort required for its imaging. The proposed developments are integrated into a computational platform based on a regularized boundary integral equation (BIE) method for 3D elastodynamics, and illustrated via a set of canonical numerical experiments.

1. Introduction

To date, inverse obstacle scattering remains a vibrant subject of interdisciplinary research with applications to many areas of science and engineering (Pike et al., 2002). Its purpose is to recover the geometric as well as physical properties of unknown heterogeneities embedded in a medium from the remote observations of thereby scattered waveforms. Such goal is pursued by studying the nonlinear and possibly non-unique relationship between the scattered field produced by a hidden object, e.g. fracture, and its characteristics.

From the mathematical viewpoint, the fracture reconstruction problem was initiated in Kress (1995) where, from the knowledge of the far-field scattered waveforms, the shape of an open arc was identified via the Newton's method. This work was followed by a suite of *non-iterative* reconstruction approaches such as the factorization method (e.g. Boukari and Haddar, 2013), the linear sampling method (LSM) (Kirsch and Ritter, 2000) and the concept of topological sensitivity (TS) (Guzina and Bonnet, 2004) that are capable of retrieving the shape, location, and the size of buried fractures. Recently, a TS-related approach has also been proposed for the reconstruction of a collection of small cracks in elasticity (Ammari et al., 2013b).

A non-iterative approach to inverse scattering which motivates the present study is that of TS (Guzina and Bonnet, 2004; Bonnet and Guzina, 2004). In short, the TS quantifies the leading-order perturbation of a given misfit functional due to the nucleation of an infinitesimal scatterer at a sampling point in the reference (say intact) domain. The resulting TS distribution is then used as an *anomaly indicator* by equating the support of its most pronounced negative values with that of a hidden scatterer. The strength of the method lies in providing a computationally efficient way of reconstructing distinct inner heterogeneities without the need for prior information on their geometry. Recently, a rigorous mathematical analysis of the TS for point-like anomalies is performed (Ammari et al., 2012, 2013a) which not only justifies the TS approach as a valid imaging algorithm, but also reveals that

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the TS, compared to other imaging functionals (e.g. MUSIC and backpropagation), is more robust both in terms of measurement and medium noise (Ammari et al., 2012). Moreover, Bellis and Bonnet (2013) demonstrated the ability of TS to image *traction-free cracks*. Motivated by the reported capability of TS to not only image – but also characterize – elastic inclusions (Guzina and Chikichev, 2007), this study aims to explore the potential of TS for simultaneous imaging and interfacial characterization of fractures with contact condition due to e.g. the presence of asperities, fluid, or proppant at their interface.

Existing studies on the sensing of obstacles with unknown contact condition reflect two principal concerns, namely: (i) the effect of such lack of information on the quality of geometric reconstruction, and (ii) the retrieval – preferably in a non-iterative way – of the key physical characteristics of such contact. The former aspect is of paramount importance in imaging stress corrosion fractures (Hernandez-Valle et al., 2014), where the crack extent may be underestimated due to interactions at its interface, leading to a catastrophic failure. To help address such a problem, Boukari and Haddar (2013) developed the factorization method for the shape reconstruction of acoustic *impedance* cracks. Studies deploying the LSM as the reconstruction tool (e.g. Cakoni and Colton, 2014; Cakoni and Monk, 2010), on the other hand, show that the LSM is successful in imaging obstacles and fractures *regardless* of their boundary condition. As to the second concern, a variational method was proposed in Colton and Cakoni (2004) to determine the essential supremum of electrical impedance at the boundary of partially-coated obstacles. By building on this approach, Cakoni and Kress (2013) devised an iterative algorithm for the identification of surface properties of obstacles from acoustic and electromagnetic data. Recently, Minato and Ghose (2014) proposed a Fourier-based algorithm using reverse-time migration and wavefield extrapolation to retrieve the location, dip and heterogeneous compliance of an elastic interface under the premise of (a) one-way seismic wavefield, and (b) absence of evanescent waves along the interface.

Considering the small-amplitude elastic waves that are typically used for seismic imaging and non-destructive material evaluation, the Schoenberg's *linear slip model* (Schoenberg, 1980) is widely considered as an adequate tool to describe the contact condition between the faces of a fracture. This framework can be interpreted as a linearization of the interfacial behavior about the elastostatic equilibrium state (Pourahmadian et al., 2012) prior to elastic-wave excitation, which gives rise to linear (normal and shear) specific stiffnesses k_n and k_s. Here it is worth noting that strong correlations are reported in the literature (Verdon and Wustefeld, 2013; Choi et al., 2014; Ahmadian et al., 2010) between (k_s, k_n) and surface roughness, residual stress, fluid viscosity (if present at the interface), intact material properties, fracture connectivity, and excitation frequency. In this vein, remote sensing of the specific stiffness ratio k_s/k_n has recently come under the spotlight in hydraulic fracturing, petroleum migration, and Earth's Critical Zone studies (Knight et al., 2010; Baird et al., 2013). By way of laboratory experiments (Choi et al., 2014; Place et al., 2014; Bakulin et al., 2000), it is specifically shown that k_s/k_n – often approximated as either one (dry contact) or zero (isolated fluid-filled fracture) – can deviate significantly from such canonical estimates, having fundamental ramifications on the analysis of the effective moduli and wave propagation in fractured media. A recent study (Baird et al., 2013; Verdon and Wustefeld, 2013) on the production from the Cotton Valley tight gas reservoir, using shear-wave splitting data, further highlights the importance of monitoring k_s/k_n during hydraulic fracturing via the observations that: (i) the correlation between proppant introduction and dramatic increase in k_s/k_n can be used as a tool to directly image the proppant injection process; (ii) the ratio k_s/k_n provides a means to discriminate between newly created, old mineralized and proppant-filled fractures, and (iii) k_s/k_n may be used to monitor the evolving hydraulic conductivity of an induced fracture network and subsequently assess the success of drilling and stimulation strategies.

In what follows, the TS sensing platform is developed for the inverse scattering of time-harmonic elastic waves by fractures with unknown geometry and contact condition in $\mathbb{R}^3$. On postulating the nucleation of an infinitesimal penny-shaped fracture with constant (normal and shear) interfacial stiffnesses at a sampling point, the TS formula and affiliated elastic polarization tensor are calculated and expressed in closed form. Simulations demonstrate that, irrespective of the contact condition between the faces of a hidden fracture, the TS is capable of reconstructing its geometry and identifying the normal vector to the fracture surface without iterations. Assuming illumination by long wavelengths, it is further shown that the TS is capable (with only a minimal amount of additional computation) of qualitatively characterizing the ratio k_s/k_n along the surface of nearly-planar fractures. The proposed developments are integrated into a computational platform based on a regularized boundary integral equation (BIE) method for 3D elastodynamics. For completeness, the simulations also include preliminary results on the "high"-frequency TS sensing of fractures with specific stiffness, which may motivate further studies in this direction.

2. Preliminaries

Consider the scattering of time-harmonic elastic waves by a smooth fracture surface $\Gamma \subset \mathcal{B}_1 \subset \mathbb{R}^3$ (see Fig. 1) with a linear, but otherwise generic, contact condition between its faces $\Gamma^{\pm}$. For instance the fracture may be partially closed (due to surface asperities), fluid-filled, or traction free. Here, $\mathcal{B}_1$ is a ball of radius R_1 – containing the sampling region i.e. the search domain for hidden fractures. The action of an *incident plane wave* $\boldsymbol{u}^{\mathrm{i}}$ on Γ results in the scattered field $\tilde{\boldsymbol{u}}$ – observed in the form of the total field

$$\boldsymbol{u}(\xi) = \boldsymbol{u}^{\mathrm{i}}(\xi) + \tilde{\boldsymbol{u}}(\xi), \qquad \xi \in S^{\mathrm{obs}}, \tag{1}$$

over a closed measurement surface $S^{\mathrm{obs}} = \partial\mathcal{B}_2$, where $\mathcal{B}_2$ is a ball of radius $R_2 \gg R_1$ centered at the origin. The reference i.e. "background" medium is assumed to be elastic, homogeneous, and isotropic with mass density ρ, shear modulus μ, and Poisson's ratio ν.

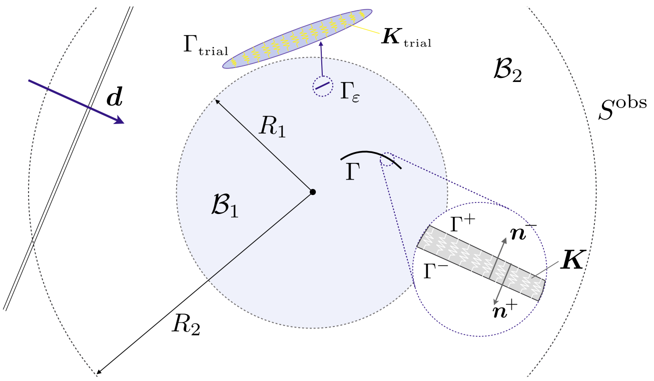

Fig. 1. Illumination of a hidden fracture $\Gamma \in \mathbb{R}^3$ with specific stiffness $\boldsymbol{K}$ by a plane (P- or S-) incident wave propagating in direction $\boldsymbol{d}$, where the induced wavefield is monitored over S^{obs}.