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Variation of stress intensity factor along a small interface crack front in singular stress fields

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ABSTRACT

Stress singularities usually occur at vertexes in three-dimensional joints. Cracks frequently initiate at the vertex, and the joint fails under an external force or a thermal load. In the present study, the stress distribution near a small crack occurring at a vertex in a three-dimensional joint under a tensile load is examined, and the stress intensity factors at the crack tip are investigated along the crack tip front. The joint is made of Si and resin. In the analysis, three kinds of crack shapes (triangular, quarter circular, and concave) are supposed as the initial crack shape. The stress distribution around the crack is normalized using the singular stress near the vertex since the crack exists in the singular stress field. The stress intensity factor varies along the crack tip from the inner point to the free surface. The value of the stress intensity factor varies following the order of the stress singularity at the point of the free surface. In this analysis, the stress intensity factor is expressed as a function of the distance from the cross point of the free surface and the crack tip. Specifically, the function is composed of an exponential function of the difference between the order of the stress singularity at the cross point and the crack tip.

1. Introduction

It is well known that singular stress fields occur at the corners of joint interfaces between dissimilar materials due to the mismatch of their material properties. When an external force or a temperature change is applied to such a joint, a crack occurs at or near the interface due to the singular stress field; this frequently results in the failure of the joint. On the stress singularity, Williams (1959) derived an exact solution for elastic solids with an interface crack and reported rapid oscillations in the stress and displacement fields.

$$[\sigma_{yy}+i\tau_{xy}]_{\theta=0}=\frac{K_1^*+iK_2^*}{\sqrt{2\pi r}}\left\{\cos\left(\varepsilon\ln\frac{r}{\ell}\right)+i\sin\left(\varepsilon\ln\frac{r}{\ell}\right)\right\}$$
$$\delta_y+i\delta_x=\frac{K_1^*+iK_2^*}{2(1+2i\varepsilon)\cosh(\varepsilon\pi)}\left\{\frac{\kappa_1+1}{\mu_1}+\frac{\kappa_2+1}{\mu_2}\right\}\left(\frac{r}{2\pi}\right)^{\frac{1}{2}}\left(\frac{r}{\ell}\right)^{i\varepsilon} \quad (1)$$
$$\varepsilon=\frac{1}{2\pi}\ln\left(\frac{1-\beta}{1+\beta}\right)$$

where β is a dimensionless composite parameter depending on the material properties introduced by Dundurs (1969).

Here, K_1^* and K_2^* are the parameters for characterizing the near tip field of the interface crack, r is the distance from the crack tip, and ℓ represents a reference length. When the two materials are identical $\varepsilon = 0$, the factors K_1^* and K_2^* becomes identical to K_{I} for the mode I (tensile opening) and K_{II} for the mode II (plane shear), respectively.

Subsequently, Sih et al. (1962) conducted an analysis of stress singularity at crack tips in plane problems and bending problems using a complex variable method. Rice and Sih (1965) proposed the stress intensity factors k_1 and k_2 for an interface crack of length $2a$ under a uniform tensile load P. For the same problem, Erdogan (1965) proposed a complex stress intensity factor. Rice (1988) reexamined elastic fracture mechanics for a crack on the interface between dissimilar solids. Hutchinson and Suo (1991) indicated that the real and imaginary parts of the complex stress intensity factor correspond to modes I & II for an ordinary crack, respectively. Miyazaki et al. (1993) analyzed the stress intensity factors of interface crack using a boundary element method. Qu and Bassani (1993) developed the fracture mechanics of interface cracks in anisotropic bimaterials and derived explicit expressions for the crack-tip fields and the associated stress intensity factors. It is very important to evaluate the stress intensity factor for a small crack at the vertex in the singular stress field for estimating the real strength of joints. Akisanya and Fleck (1997) performed

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stress analysis for a crack at the interface edge in a two-dimensional joint and demonstrated that the stress intensity factor became large for a short crack due to the effect of singular stress fields and reduced for a long crack. Bjerken and Persson (2001) presented a method for obtaining the complex stress intensity factor for an interface crack in a bimaterial using a minimum number of computations. Ioka et al. (2002) conducted stress analysis in a two-dimensional joint with a small crack at the edge of the interface and investigated the effect of the singular stress field on the stress at the crack tip. Sukumar et al. (2004) presented a method for determining the complex stress intensity factor for an interface crack in two-dimensional joints using an enriched finite element. Leguillon et al. (2003) an Irwin-like criterion was used to examine the failure of two steel plates bonded together by an epoxy joint. They discussed several criterions for crack initiation at sharp notches and tried to propose a simple and robust criterion to predict brittle crack initiation at bond ends. Johnson and Qu (2007) proposed a method for calculating the stress intensity factor of an interface crack under a temperature gradient by evaluating the path-independent integral around the crack tip region using the finite element method. Recently, Hirai et al. (2012) presented an extension of the Stroh formalism and the *H*-integral derived from Betti's reciprocal principle for piezoelectric problems. They proposed a new definition of the SIFs of an interfacial corner for piezoelectric materials. Hwu and Huang (2012) presented a definition of the stress intensity factors for interface corners including interface cracks. On three-dimensional singularities and cracks, Lee et al. (1987) calculated the stress intensity factors for a crack of arbitrary planar shape near a bimaterial interface using the body-force method. Ghahremani and Shih (1992) investigated singular stress fields near the intersection of a planar interfacial crack with the free surface in joined materials. Nakamura and Kamath (1992) conducted a three-dimensional finite element analysis of the mechanics of crack growth and decohesion in a highly compliant thin film bonded to a rigid substrate. Gosz et al. (1998) developed new domain integrals for extracting mixed-mode stress intensity factors along curved three-dimensional bimaterial interface cracks. In the derivation, the asymptotic auxiliary fields for the plane problem of a bimaterial interface crack were imposed along a curved crack front. Begley and Ambrico (2001) presented the mechanics governing delamination of thin films driven by thermal expansion mismatch from two-dimensional interface flaws. Results were presented for energy release rates and mode-mixity along two-dimensional crack fronts. Song and Wolf (2002) developed a scaled boundary finite-element method, a semi-analytical boundary-element method based on finite elements, for analyzing fracture mechanics problems. Zhou et al. (2005) proposed a new three-dimensional variable-order singular boundary element for stress analysis of three-dimensional interface cracks and internal material junctions. Ortiz and Cisilino (2005) presented a general numerical tool for the analysis of three-dimensional bimaterial interface cracks. Ayhan et al. (2006) developed an efficient computational technique that utilizes enriched crack tip elements containing the correct interface crack tip asymptotic behavior. In the enriched element formulation, the stress intensity factors K_{I}, K_{II}, and K_{III} were treated as additional degrees of freedom and were obtained directly during the finite element solution phase. Tvergaard and Hutchinson (2008) investigated the effect of modes I, II, and III on interface crack growth. Chiu and Lin (2009) investigated the variation in the stress intensity factor, energy release rate, and phase angle along the front of a crack using a virtual growth method for a three-dimensional interface crack in electronic devices. Nagai et al. (2007) calculated the stress intensity factor for a three-dimensional interface crack in dissimilar anisotropic materials. Recently, Veluri and Jensen (2013) presented a method for determining the shape of a growing interface crack between a thin film and a substrate by means of a commercial finite element program.

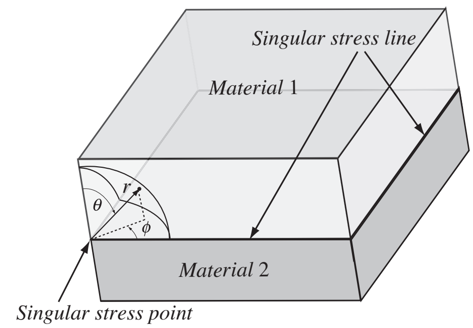

Fig. 1. Singular stress point and singular line in a three-dimensional joint.

There are few papers on three-dimensional interface cracks under a complicated stress state such as the singular stress field. The authors investigated the characteristics of singular stress fields at the vertex of the interface in three-dimensional joints (Koguchi et al., 2012) and in particular, the relationship between singular stress lines and a three-dimensional singular stress field (Koguchi and da Costa, 2010). In the three-dimensional stress singularity field, the stress fields can be expressed as

$$\sigma_{ij}(r,\theta,\phi) = \sum_{l=1}^{m} K_l f_{lij}(\theta,\phi) r^{-\lambda_l} + K_{m+1} f_{(m+1)ij}(\theta,\phi) + K_{m+2} f_{(m+2)ij}(\theta,\phi) \ln r \tag{2}$$

where K_l represents the lth intensity of the singularity, $f_{lij}(\theta,\phi)$ is the angular function of the lth value of the stress singularity, and r is the distance from the singular stress point (see Fig. 1).

Cracks frequently initiate at the singular stress point of the vertex in three-dimensional joints. Koguchi and Kimura (2014) calculated the energy release of a small crack at an initial stage of crack occurrence in a singular stress field. In several papers (Nagai et al., 2007; Kuo and Hwu, 2010) on three-dimensional interface cracks, the definition of the stress intensity factor for the generalized plane strain condition on the basis of the two-dimensional anisotropic elasticity was employed. In the present paper, a stress intensity factor corresponding to a complex angular function for a complex singularity is defined, and the angular function is normalized by the values at the interface of the angular function. This means that the stress intensity factor of the interface crack is defined using angular functions for a three-dimensional crack. The stress intensity factor will be investigated along the crack front. Additionally, an expression for the variation in the stress intensity factor against the distance from a singular point, which is the intersection of the crack front with the free surface, will be derived, and its validity will be demonstrated by comparing the two results.

2. Method for analysis

In the present paper, the stresses and displacements in a three-dimensional elastic body will be calculated using a boundary integral equation,

$$u_i(q) = \int_{\Omega} \left[U_{ij}(q,Q) t_j(Q) - T_{ij}(q,Q) u_j(Q) \right] ds(Q) \tag{3}$$

where q is an inner point in the domain Ω, Q is a point on the boundary Γ, and U_{ij} and T_{ij} are the fundamental solutions for the displacement and traction, respectively. Here, Rongved's solution,