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A constitutive framework for the elastoplastic modelling of geomaterials

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ABSTRACT

The object of this paper is the definition of a generalised constitutive model for geomaterials, based on elastoplasticity. Two are the tasks of the paper: the substantial enhancement of an existing hierarchical yield function and the definition of an isotropic hardening constitutive model for the behaviour of both cohesive and frictional geomaterials. After its definition, the new function will be compared with different yield surfaces existing in literature in both the deviatoric and the meridian representation. Thereafter, the constitutive model will be outlined by defining hierarchical expressions for hardening laws and stress–dilatancy relationship in order to take into account several aspects of soil behaviour. Finally, some comparisons between data from experimental tests on clay and sand will be made to highlight the hierarchical structure of the proposed model.

1. Introduction

The Modelling of the behaviour of cohesive and frictional materials is often treated with different approaches and many models can be found in the geotechnical literature, all describing clays and sands as two classes of materials. Historically, such a distinction is evidenced by the formulation of Cam-Clay-like models (Nakai and Matsuoka, 1986; Bardet, 1990; Kavvadas and Amorosi, 2000; Rouainia and Wood, 2000; Vatsala et al., 2001; Liu and Carter, 2002; Dafalias et al., 2006; Suebsuk et al., 2010) and extended Mohr–Coulomb-like models (Ghaboussi and Momen, 1982; Prévost, 1985; Muir Wood et al., 1994; Manzari and Dafalias, 1997; Gajo and Muir Wood, 1999; Li and Dafalias, 2000; Li, 2002; Loukidis and Salgado, 2009; Lashkari, 2010). Other approaches, instead, have been formulated to cover the general behaviour of geomaterials in a unique framework (Desai et al., 1986; Pestana and Whittle, 1999; Pestana et al., 2002a,b; Yao et al., 2008).

This paper may be included in the latter category as it pursues the goal to define a starting platform for modelling different peculiar aspects related to soil behaviour. Defining a unique constitutive platform is important not only from the conceptual point of view, but also because it allows to simplify implementation into numerical codes. Fulfilling this task is clearly not simple because, as stated by Muir Wood (1990), "*The more effects there are to be built into the model, the more elaborate that model becomes, and the more soil parameters are required to specify the model. The more parameters that are required, the more complex the laboratory testing that is needed to determine their values becomes*". These aspects are self-evident in models that act as unification of simpler plasticity models (Brannon et al., 2009).

On the other hand, constitutive modelling of geomaterials should include many features as non-linear elasticity, pressure sensitivity, non-associated flow, inherent and induced anisotropy, void ratio dependency, cyclic phenomena, time dependence and many others. These features, however, should be introduced hierarchically on the basis of the experimental tests available for the determination of parameters. In a word, models should be "advanced" desirably without the shortcomings listed by Kolymbas (2000).

It is clear that a generalised model for soils based on elastoplasticity requires the assumption of "powerful" expressions of yield surfaces and plastic potentials in principal stress space as well as generalised hardening laws. As for surfaces, their formulation must be such as to prevent the occurrence of "false elastic domains" (Brannon and Leelavanichkul, 2010), which leads to numerical problems in integration algorithms (Bier and Hartmann, 2006). As far as the hardening laws are concerned, generalised functions representative of both volumetric and deviatoric hardening should be introduced.

The model that will be presented accounts for hyperelastic behaviour and void ratio state dependency but the analysis is restricted to single surface plasticity, isotropic hardening and time independent behaviour. This version of the model is intended as the basis for future extensions to the analysis of anisotropy, material degradation, cyclic loading and time-dependent behaviour.

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2. New yield and plastic potential functions

The proposed function is based on a previous formulation (Mortara, 2010) and can be put in the form

$$\Phi = q^2 - \Phi_d \Phi_c \Phi_\rho^2 \tag{1}$$

Function Φ is written in terms of deviatoric stress q, mean stress p and Lode angle θ, which are defined as:

$$q = \sqrt{\frac{3}{2} s_{ij} s_{ij}} \quad p = \frac{1}{3}\sigma_{kk} \quad \theta = \frac{1}{3}\arccos\left(\frac{9}{2}\frac{s_{ij}s_{jk}s_{ki}}{q^3}\right) \tag{2}$$

being σ_{ij} the Cauchy stress tensor and $s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$ the deviatoric stress tensor (where δ_{ij} is the Kronecker delta). Note that, for the generality of the model, the analysis is not restricted in terms of the effective stress tensor σ'_{ij}. However, when used for soils, the constitutive law is meant to be formulated in terms of effective stresses. Product $\Phi_d\Phi_c\Phi_\rho^2$ defines the hierarchical structure of Eq. (1) and rules the shape of the yield surface in meridian and deviatoric planes. Functions Φ_d and Φ_c define the shape of the surfaces in the meridian representation:

$$\Phi_d = S_p \alpha_d R_p^{n_\alpha} \tag{3}$$

$$\Phi_c = 1 - (\text{sgn}\, n_c) S_p R_p^{n_c} \tag{4}$$

where S_p and R_p are given by

$$S_p = (1 - k_R)\text{sgn}(p_t + p) - k_R \text{sgn}(p - p_c) \tag{5}$$

$$R_p = (1 - k_R)\left|\frac{p_t + p}{p_t + p_c}\right| + k_R\left|\frac{p - p_c}{p_t + p_c}\right| \tag{6}$$

being k_R a flag parameter that takes the value 0 or 1. Due to the definition of the cap function Φ_c, if $n_c = 0$ the surface $\Phi = 0$ appears as an open surface centred to the hydrostatic axis with linear or curved generatrixes. This is one of the features not included in the previous formulation (Mortara, 2010). In the above equations, n_α and α_d are given by:

$$n_\alpha = [1 - \text{sgn}\, n_c]2n_d + \frac{n_c[1 - \text{sgn}\, n_c + R]^{n_c}}{1 - (\text{sgn}\, n_c)[1 - \text{sgn}\, n_c + R]^{n_c}} \tag{7}$$

$$\alpha_d = \frac{[1 - \text{sgn}\, n_c] + (\text{sgn}\, n_c)R^{2-n_\alpha}}{1 - (\text{sgn}\, n_c)[1 - \text{sgn}\, n_c + R]^{n_c}}[S_R \eta_h (p_t + p_c)]^2 \tag{8}$$

where R and S_R are defined as

$$R = (1 - k_R)R_h + k_R(1 - R_h) \tag{9}$$

$$S_R = 1 - (\text{sgn}\, n_c)k_R\left(2 - \frac{1}{1 - \text{sgn}\, n_c + R}\right) \tag{10}$$

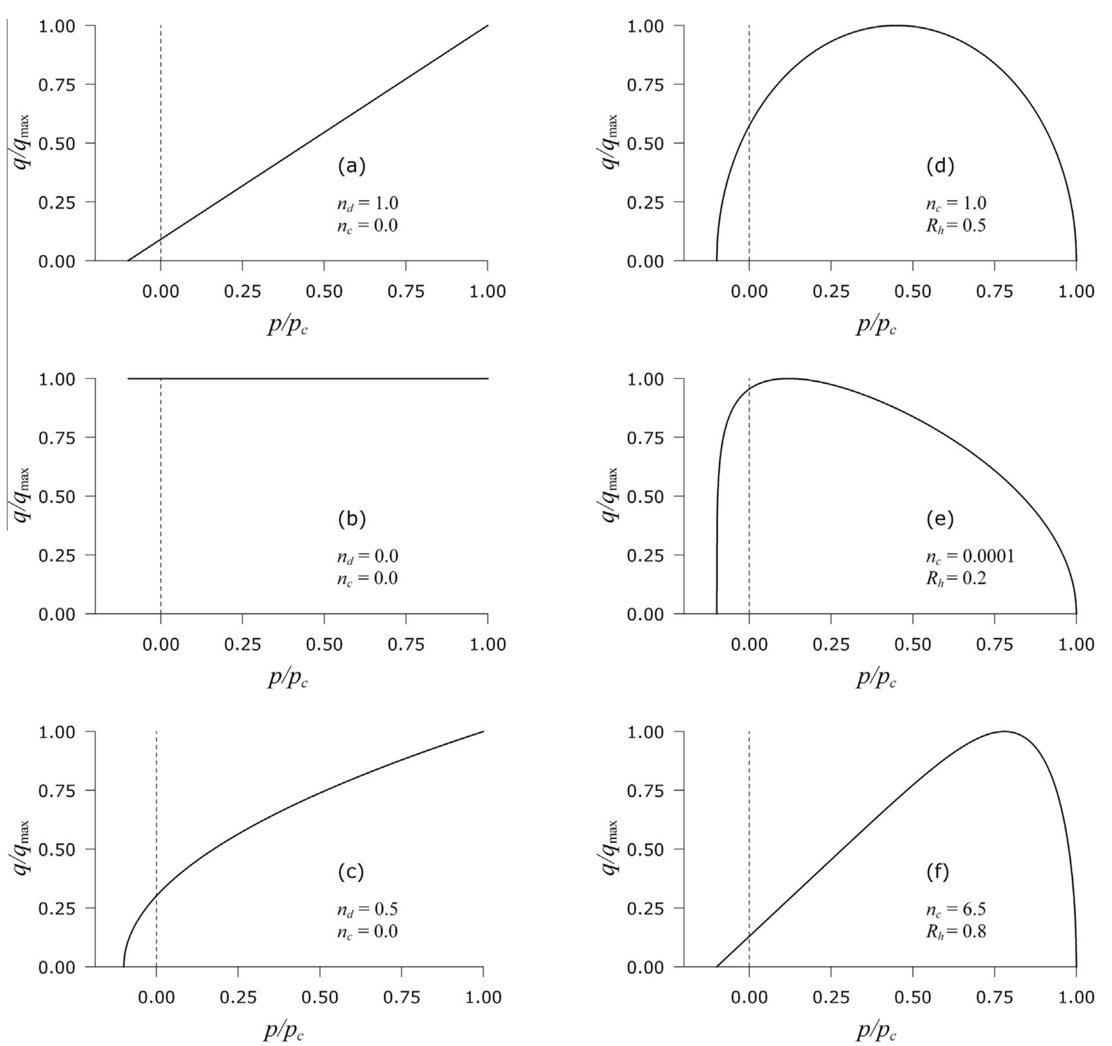

Fig. 1. Selected shapes of surface $\Phi = 0$ for $\Phi_\rho = 1$: (a) cone, (b) cylinder, (c) paraboloid, (d) ellipsoid, (e) bullet, (f) tear.