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A contact problem in couple stress thermoelasticity: The indentation by a hot flat punch

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ABSTRACT

It is well known, that thermo-elastic effects may have significant results upon the macroscopic response in the mechanics of contact. On the other hand, as the scales in the contact system reduce progressively (micro to nano-scales), the internal material lengths become important and their effect upon the macroscopic response cannot be ignored. The present work extends the classical contact solution for a hot flat punch indenting a homogeneous elastic half-plane, where heat conduction is permitted (Comninou et al., 1981), to the analogous case of an indented microstructured solid. The behavior of the indented material is modeled through the couple-stress elasticity theory, which introduces characteristic material lengths and is appropriately modified in order to incorporate the thermal effects. The problem formulation is based on singular integral equations, resulted from a treatment of the mixed boundary value problems via integral transforms and generalized functions. The results show significant departure from the predictions of classical thermoelasticity showing that the microstructural characteristics of the material should not be ignored.

1. Introduction

The contact of two bodies maintained at different temperatures yields to thermo-elastic deformations at the contact region that, although small, can affect the contact pressure distribution and, depending on the temperature difference between the two bodies, even the contact area. Assuming that the heat flows only through the contact area and that no heat flows across the exposed surfaces, theoretical investigations by Barber (1971, 1973, 1978) on indentation problems predict that the regions near the contact area expand when the indentor's temperature is raised over a specific limit, causing the separation of the two solids, if the compressive load is maintained constant. This separation is expected to cause a reduction in the extent of the contact area between the indentor and the indented elastic body. This behavior was also experimentally confirmed by Clausing (1966) who, almost ten years earlier, had shown that the thermal contact resistance between two contacting bodies varies with the transmitted heat flux as a result of the thermo-elastically driven changes in the extend of the contact area.

It is well-known that material microstructure influences the macroscopical behavior of complex materials, such as composites, cellular materials and ceramics. In fact Maranganti and Sharma (2007) showed that gradient effects play significant role in complex materials with course-grain microstructure, while Chen et al. (1998) developed a continuum model for cellular materials and concluded that the continuum description of this class of materials obeys a gradient elasticity theory of the couple-stress type by naturally identifying the cell size with the material length scale. Size effects have been also predicted for two dimensional grid-works (Askar and Cakmak, 1968) and three dimensional cubic lattices (Lakes, 1986) and, associated with Cosserat elasticity, lead to an increase in moduli with decreasing specimen size relative to the cell size (Onck et al., 2001). Finally, strain gradient effects, even though difficult to be measured, have been observed in rigid polyurethane and polymethacrylimide foams (Lakes, 1986; Anderson and Lakes, 1994).

While classical continuum theories do not incorporate internal length-scales and therefore cannot take into account micromechanical effects, the use of generalized continuum theories (see e.g. Maugin, 2010) allows to achieve a more effective description of the mechanical response when, for instance, stress concentrations appear (Georgiadis, 2003; Gourgiotis and Piccolroaz, 2014) or instability phenomena are involved (Dal Corso and Willis, 2011; Bacigalupo and Gambarotta, 2013). The need for such generalized continuum models has also been verified through experimental (Lakes et al., 1985; Beveridge et al., 2013) and

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theoretical (Smyshlyaev and Fleck, 1995; Bigoni and Drugan, 2007; Bacca et al., 2013a,b; Bacigalupo, 2014) approaches.

During indentation, size effects can be dominant especially when the indentation size is comparable to the material microstructure. This process has been modeled employing classical theories by directly incorporating the microstructural characteristics into the model through purely geometrical considerations (see e.g. Chen et al., 2004; Stupkiewicz, 2007; Fleck and Zisis, 2010; Zisis and Fleck, 2010) and phenomenological approaches based on gradient elasticity/ plasticity ideas, or on discrete dislocation concepts (Muki and Sternberg, 1965; Poole et al., 1996; Begley and Hutchinson, 1998; Nix and Gao, 1998; Shu and Fleck, 1998; Wei and Hutchinson, 2003; Danas et al., 2012; Zisis et al., 2014). Even though, purely elastic indentation of materials is hard to achieve in practice (Larsson et al., 1996), elasticity can be of interest in particular cases. In fact, there are materials, such as polymers, that exhibit significant size effects also in the elastic regime (Han and Nikolov, 2007; Nikolov et al., 2007).

In the present study, the steady-state plane-strain contact problem of the hot frictionless flat punch indenting a couple-stress elastic half-plane is investigated for the first time to analyze the influence of the internal length scale upon the macroscopic response. In addition to the dependence of the response upon the heat flux amount from the indentor to the substrate and the magnitude of the indentation load observed in the classical framework, it is shown that the type of contact (perfect contact throughout the width of the indentor or separation near the corners of the punch) occurring is strongly affected by the microstructural characteristics of the material. The opposite problem of a cool flat punch indenting an elastic half-plane with microstructure, characterized by the possibility of having imperfect contact (Barber, 1971, 1973, 1978; Comninou and Dundurs, 1979), will be a subject of a future work.

The paper is organized as follows. In Sections 2 and 3 the fundamental equations of couple-stress thermoelasticity are summarized and particularized to plane-strain case. In Section 4 the problem of the indentation of an elastic half-plane by a hot flat punch is formulated and the appropriate boundary conditions are described. The mixed boundary value problem is attacked via Fourier transforms and singular integral equations (Sections 5 and 6). Accordingly, the integral equations are solved by employing analytical and numerical considerations in Section 7. The results are discussed in detail in the final part.

The attained results have genuine practical application in qualitatively identifying the influence of length scale effects in solids, a requirement of practical importance for the advanced design of materials and structures.

2. Fundamentals of couple-stress thermoelasticity

One of the most effective generalized continuum theories is that of couple-stress elasticity, also known as Cosserat theory with constrained rotations (Mindlin and Tiersten, 1962; Koiter, 1964). In this theory, the modified strain-energy density and the resulting constitutive relations involve, besides the usual infinitesimal strains, certain strain gradients known as the rotation gradients. The generalized stress–strain relations for the isotropic case include, in addition to the conventional pair of elastic constants, two new elastic constants, one of which is expressible in terms of a material parameter that has dimension of [length]. The presence of this length parameter, in turn, implies that the modified theory encompasses the possibility of size effects. This theory was extended by Nowacki (1966) who derived constitutive equations on the basis of thermodynamics of irreversible processes and provided the fundamental differential equations of couple-stress thermoelasticity.

We begin by giving an account of the theory of couple-stress thermoelasticity as introduced by Nowacki (1966). In the absence of inertia effects, the balance laws for the linear and angular momentum lead to the following force and moment equations of equilibrium (Mindlin and Tiersten, 1962)

$$\sigma_{ji,j} + X_i = 0, \tag{1}$$

$$e_{ijk}\sigma_{jk} + \mu_{ji,j} + Y_i = 0, \tag{2}$$

where a Cartesian rectangular coordinate system $Oxyz$ is used along with indicial notation and summation convention. In these equations σ_{ij} is the force-stress tensor, μ_{ij} is the couple-stress tensor, X_i denotes components of the body-force vector referred to a body unit, and Y_i denotes the components of the body-couple vector, a comma denotes partial differentiation and e_{ijk} is Levi–Civita alternating symbol. Further, σ_{ij} can be decomposed into its symmetric and anti-symmetric components as follows

$$\sigma_{ij} = \tau_{ij} + \alpha_{ij} \tag{3}$$

with $\tau_{ij} = \tau_{ji}$ and $\alpha_{ij} = -\alpha_{ji}$, whereas it is advantageous to decompose μ_{ij} into its deviatoric $\mu_{ij}^{(D)}$ and spherical $\mu_{ij}^{(S)}$ parts in the following manner

$$\mu_{ij} = m_{ij} + \frac{1}{3}\delta_{ij}\mu_{kk} \tag{4}$$

with $\mu_{ij}^{(D)} = m_{ij}$, $\mu_{ij}^{(S)} = (1/3)\delta_{ij}\mu_{kk}$, and δ_{ij} is the Kronecker delta. Now, with the help of the Green-Gauss theorem and employing the moment equation of equilibrium (2), one may obtain the anti-symmetric part of the stress tensor as

$$\alpha_{ij} = -\frac{1}{2}e_{ijk}\left(\mu_{pk,p} + Y_k\right) \tag{5}$$

from which follows that the stress tensor is symmetric in the absence of body couples and for a vanishing divergence of couple-stresses. Finally, combining (1)–(5) yields the final equation of equilibrium which involves only the symmetric part stress tensor and the deviatoric part of the couple-stress tensor

$$\tau_{ji,j} - \frac{1}{2}e_{jki}(m_{pj,pk} + Y_{j,k}) + X_i = 0. \tag{6}$$

Concerning the kinematical description of the continuum, the following primary kinematical fields are defined in the framework of the geometrically linear theory

$$\varepsilon_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j}), \quad \omega_i = \frac{1}{2}e_{ijk}u_{k,j}, \quad \kappa_{ij} = \omega_{j,i}, \tag{7}$$

where ε_{ij} is the strain tensor, ω_i is the rotation vector, and κ_{ij} is the curvature tensor (i.e. the gradient of rotation or the curl of the strain) expressed in dimensions of $[\text{length}]^{-1}$, which by definition is traceless: $\kappa_{ii} = 0$ since $\omega_{i,i} = 0$. Accordingly, the compatibility equations for the kinematical fields in (7) are (Naghdi, 1965)

$$e_{ipm}e_{mjk}\varepsilon_{ij,k} + e_{pjm}\kappa_{mj} = 0, \quad e_{ikm}e_{jpm}\kappa_{ij,k} = 0. \tag{8}$$

where the elimination of κ_{ij} between (8) leads to the usual Saint Venant's compatibility equations for the strain tensor components.

Regarding the boundary conditions, we note that in the constrained couple-stress theory the normal component of the rotation vector is fully specified by the distribution of tangential displacements over the boundary. This implies that the traction boundary conditions, at any point on a *smooth* boundary or section, consist of the following three *reduced* force-tractions and two *tangential* couple-tractions (Mindlin and Tiersten, 1962; Koiter, 1964)

$$P_i^{(n)} = \sigma_{ji}n_j - \frac{1}{2}e_{ijk}n_j m_{(nn),k}, \tag{9}$$