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Bounding surface modeling of sand with consideration of fabric and its evolution during monotonic shearing

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ABSTRACT

This paper presents a bounding surface plasticity model for sand that considers fabric and its evolution during monotonic shearing. The model is based on critical-state soil mechanics. The bounding surface controls sand stiffness through a relationship that depends on the distance from the current state to the bounding surface calculated using a rigorous algorithm. Dilatancy, which measures the plastic volume change caused by plastic shear deformation, is captured through a newly introduced phase transformation line. The fabric is quantified based on the distribution of contact normals between particles; it affects the location of the phase transformation line (thus, the dilatancy). Simulation results using the model are in excellent agreement with test data for Toyoura sand.

1. Introduction

The response of sand, a particulate (granular) material, to loading has some key features: (1) highly non-linear stress–strain response (Carraro et al., 2004; Mitchell and Soga, 2005; Murthy et al., 2007; Salgado, 2008), (2) achievement of a critical (steady) state upon substantial shearing (Casagrande, 1936; Mitchell and Soga, 2005; Salgado, 2008; Schofield and Wroth, 1968), (3) dilatancy (plastic volumetric change caused by plastic shear deformation) (Bolton, 1986; Chakraborty and Salgado, 2009; De Josselin de Jong, 1976; Rowe, 1962; Salgado, 2008; Zhang and Salgado, 2010) and (4) dependence on fabric, which represents how particles assemble (Oda, 1972; Oda et al., 1978). A constitutive model should capture these properties if it is to describe the mechanical response of sand properly.

Bounding surface models for sand (Dafalias and Manzari, 2004; Li and Dafalias, 2012; Li, 2002; Loukidis and Salgado, 2009; Manzari and Dafalias, 1997; Taiebat and Dafalias, 2008) have captured the highly nonlinear stress–strain relationship of sand and its achievement of a critical state successfully. If soil is in a critical state, the stress state and density of the soil no longer change, even if shearing continues. The locus of critical states in stress space is the critical state surface. The bounding surface (also a surface in stress space) conceptually bounds every possible stress state that the soil may reach. In these models, the bounding surface is initially larger than the critical state surface. As loading proceeds, the bounding surface shrinks towards the critical state surface; consequently, the final destination of the stress state is on the critical state surface.

There are generally two ways for the bounding surface to bound every possible stress state during loading: enforcement of the consistency condition on the bounding surface (Li, 2002) or numerical enforcement of the concept by having the hardening modulus take negative values when the stress steps outside the bounding surface (Dafalias and Manzari, 2004; Li and Dafalias, 2012; Loukidis and Salgado, 2009; Manzari and Dafalias, 1997). In the bounding surface models following this second approach, the stress state can exist marginally outside the bounding surface, but, as soon as it steps outside it, the negative hardening modulus moves it back towards the bounding surface. Generally, in this type of bounding surface model, the hardening modulus is proportional to the distance between the stress state and the bounding surface, which can itself take either negative or positive values; thus, rigorous determination of this distance is important to estimate material stiffness. In this study, the distance to the bounding surface is calculated by identification of an image stress state on the bounding surface corresponding to the current stress state and the current loading direction. Woo and Salgado (2014) proposed a rigorous algorithm to determinate the image point on the bounding surface that is valid under multi-axial conditions. The present constitutive model follows that algorithm.

For a wide range of possible initial states, sand experiences contraction until it reaches the phase transformation point, after

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which it dilates with further shearing. Proper determination of when phase transformation occurs is therefore a key factor in the description of the dilatancy response in constitutive modeling. In bounding surface models for sand (Dafalias and Manzari, 2004; Li and Dafalias, 2012; Loukidis and Salgado, 2009; Manzari and Dafalias, 1997), phase transformation has been captured using a dilatancy surface in stress space. In these models, the dilatancy surface corresponds conceptually to the phase transformation points. The dilatancy surface has certain limitations when the rigorous definition of the image point used in this study is adopted, including an inability to simulate certain complex loading patterns. These limitations exist because, unlike the bounding surface, the dilatancy surface does not bound the stress state. This means that it is possible for a stress state to be reached outside the dilatancy surface from which certain loading directions will produce no image point on (no intersection with) the dilatancy surface under multi-axial conditions. This limitation may not exist or may be rare if projection rules other than the one adopted here are used, but, for the purposes of this paper, a dilatancy surface is not used. Instead, the present constitutive model estimates dilatancy using a newly-introduced phase transformation line. The dilatancy of sand is determined based on the location of the current state with respect to the phase transformation line.

The fabric, how particles assemble, has an important role in the mechanical response of sand. For example, according to Oda (1972), sand specimens prepared using different methods (each producing a different fabric) show significantly different mechanical response, all other things being equal. In the present study, fabric is quantified through a tensor defined based on the distribution of the unit vectors normal to the plane of contact between any two particles. The fabric effect factor, which is defined as the double-dot product between an incremental loading direction and the fabric tensor, represents the interaction between loading and fabric; it is used in the present constitutive model to consider the effect of fabric on dilatancy. The model also allows fabric to evolve during loading.

The present paper lays out the fundamental concepts on which the present constitutive model is based in Section 2. Section 3 describes the constitutive model formulation, including the hardening and flow rules. Section 4 shows the model performance in simulating the results of experiments with Toyoura sand. Section 5 summarizes the contributions of the present paper. The paper uses geo-mechanics sign convention (compression, contraction positive).

2. Fundamental concepts

2.1. The critical (steady) state

At the critical state, a state that a particulate material eventually reaches if sheared far enough, the following holds for a saturated sand loaded in triaxial compression or extension:

$$\dot{p}' = 0, \quad \dot{q} = 0, \quad \dot{e} = 0, \quad \dot{\varepsilon}_q \neq 0 \tag{1}$$

where p' is the mean effective stress ($= \sigma'_{kk}/3 = \sigma_{kk}/3 - u$, where u is the pore-water pressure), q (= $(3J_2)^{1/2}$) is a representation of the octahedral shear stress, e is void ratio, ε_q is a representation of the octahedral shear strain, and the dots above the variables represent time differentiation. Expressions (1) state that stress and volume do not change once sand reaches the critical state under monotonic loading conditions; thus, the critical state can be defined using stress (p' and q) and volume (e).

Fig. 1 shows the locus of critical state in p'–q–e space. In undrained (constant-volume) shearing, sand generally crosses a phase transformation state, at which dp' changes sign, before it

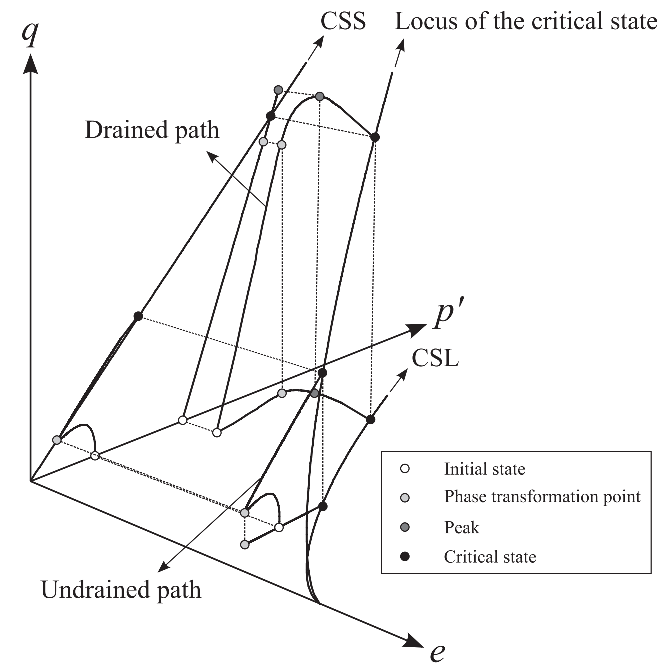

Fig. 1. The critical state line (CSL), critical state surface (CSS), and undrained and drained loading paths in p'–q–e space.

reaches the critical state. In drained (du = 0) shearing, an ultimately dilative sand first contracts, goes through a state at which the volumetric strain increment dε_v changes sign (which is not the phase transformation state, as discussed later in the paper), then dilates on its way to the critical state. Its shear stress q peaks at a value greater than the critical-state shear stress q_{cs} before the sand softens to a stable critical-state value. Regardless of the drainage conditions, the final destination of the sand is the critical state; thus, in the development of constitutive models for sand, it is essential to express the critical state in a mathematically rigorous manner.

According to Li (1997), the critical state line (CSL in Fig. 1) for sand in the e–p' plane can be expressed as:

$$e_{cs} = \Gamma_c - \lambda (p'/p_A)^{\xi} \tag{2}$$

where e_{cs} is the critical-state void ratio; p_A is the atmospheric pressure (≈ 100 kPa); Γ_c is the critical-state void ratio at p' = 0; λ and ξ are material constants. In the formulation of constitutive response, it has often been assumed (Dafalias et al., 2004; Loukidis and Salgado, 2009; Papadimitriou et al., 2005) that the CSL in the e–p' plane depends on the fabric of the sand and the loading direction. Li and Dafalias (2012) argued, using thermodynamics, that the CSL is unique in the e–p' plane; experimental results from Yoshimine and Kataoka (2007) provide evidence that the CSL in e–p' space is the same in triaxial compression and extension. While we recognize that the uniqueness of the CSL in e–p' space is not yet definitively proven, we believe that the evidence from high-quality testing does provide support for this uniqueness, and the present constitutive model assumes that the critical-state line in e–p' plane is unique; specifically, it assumes that the CSL does not depend on initial fabric or loading direction.

The critical-state surface (CSS in Fig. 1) in stress space is commonly defined by $q_{cs} = M_c p'_{cs}$, where q_{cs} is the critical-state shear stress; p'_{cs} is the critical-state mean effective stress; and M_c is the critical-state stress ratio. As p'_{cs} can be determined from the projection of the CSL on the e–p' plane, the only parameter required for estimation of q_{cs} is M_c. For particulate materials, M_c depends on the loading direction; M_{cc}, the value of M_c in triaxial compression,