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Identifying hyperelastic and isotropic materials by examining the variation of principal direction of left Cauchy–Green deformation tensor in uniaxial loading ☆

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ABSTRACT

Many bodies of biological and engineering interest have a fibrous and layered structure. Hence, it is believed that these bodies are inhomogeneous and made up of anisotropic material. Classical mechanical experiments used to find the required material symmetry in the constitutive relation cannot distinguish inhomogeneous bodies made of isotropic material and homogeneous bodies made of anisotropic material. Therefore, it is of interest to find an alternative hypothesis so that inhomogeneity and anisotropy can be determined independent of the other. This study finds that the principal (or eigen) direction of the left Cauchy–Green deformation tensor, **B** does not vary with the magnitude of the applied uniaxial load at a given location whenever the body – homogeneous or inhomogeneous – is made of isotropic and hyperelastic material and the deformations are measured from a stress free reference configuration. In general, the principal direction of the left Cauchy–Green deformation tensor varies with the magnitude of the uniaxial load when the body is made up of anisotropic material. Thus, it is concluded that if the variation in the principal direction of **B** with the magnitude of the applied uniaxial load is experimentally investigated then one could ascertain whether the body is made up of isotropic or anisotropic material.

1. Introduction

Many engineering materials have a fibrous microstructure. It is also known that (Fung, 1990, 1993; Holzapfel, 2000) many biological tissues have a layered and a fibrous structure. Hence, it is believed that these materials have to be modeled as inhomogeneous bodies made of anisotropic material (Holzapfel, 2000). However, a systematic methodology using experimental observations from mechanical experiments to decide whether these bodies need to be approximated as homogeneous bodies made of anisotropic material or inhomogeneous bodies made of isotropic material[1] or inhomogeneous bodies made of anisotropic material is absent in the literature.

Traditionally to identify anisotropy from mechanical experiments one examines whether the response of the body under investigation changes with the direction of the applied uniaxial load or by how much the ratio of the applied normal stress in the (say) x and y directions in the equal biaxial experiment differ from 1 (Strumpf et al., 1993). Following Saravanan and Rajagopal (2005), it is known that the change in the response with the direction of the applied load would happen even in case of inhomogeneous bodies made of isotropic material. Also, we show that in case of inhomogeneous bodies made of isotropic material subjected to equal biaxial experiment, the ratio of the nominal stress in the x and y direction would differ from 1, provided the material parameters vary along two directions. Only in case of homogeneous bodies whose deformations are measured from a stress free reference configuration would a change in response with the direction of the applied uniaxial load imply that the body is made of anisotropic material. Thus, the classical mechanical experiment to identify material symmetry assumes that the tested body is homogeneous. While it may be possible to make homogeneous bodies out of man made materials and test them for material symmetry, in case of naturally occurring bodies one can test only what is available. Therefore, it would be of value if experimental

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[1] To be clearer, when we say inhomogeneous body is made of isotropic material, we mean that the inhomogeneous body is made of different constituents which are isotropic.

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observations can reveal the material symmetry during mechanical testing independent of whether the body being tested is homogeneous or inhomogeneous.

It is known (Truesdell and Noll, 1965) that the symmetry group of the material that a homogeneous body is made up of depends on the configuration in which the body is in. For simple materials Noll's rule (Noll, 1958) tells how the symmetry group changes with the configuration of the homogeneous body. Consistent with the prevalent usage, a material is said to be isotropic if its symmetry group in a configuration of the homogeneous body, probably the undistorted (unstressed) state, coincides with the proper orthogonal group. Similarly, it also known that (Truesdell and Noll, 1965) if there exist a configuration for the entire body such that the mechanical response of any arbitrary subpart from this configuration is identical, then the body is said to be homogeneous. A body that is not homogeneous is said to be inhomogeneous. This definition for homogeneous body allows two distinct classes of inhomogeneous body. Different subparts of a body having different chemical composition and hence mechanical response is one class of inhomogeneous bodies. In another class, all subparts of the body have the same chemical composition but there exist no configuration in which the state of stress in the body would be uniform; in other words, in this case the internal structure variation causes a change in the mechanical response. Example of this class of inhomogeneous bodies where the state of stress is non-uniform in any configuration is residually stressed bodies; bodies which have stresses in the interior even though there is no boundary traction. Of course, a body could be inhomogeneous for both the above reasons, variation in the chemical composition and internal structure.

As discussed above, even though in mechanics the material symmetry and inhomogeneity have a precise meaning, there is a lot of ambiguity in application of these definitions to obtain constitutive relations for engineered and naturally occurring bodies. While some issues related to identifying whether a given body is homogeneous or inhomogeneous is discussed in Saravanan (2014), this article addresses the problems in finding material symmetry.

When one talks about material symmetry in continuum mechanics, one refers to the symmetry that the constitutive relation should have. It is customary to require that the set of rotations of the reference configuration that leaves the mechanical response unchanged should not alter the functional form of the constitutive relation also. There is a conundrum here. While the constitutive relation is for a material point, symmetry restriction on this constitutive relation at a point depends on the unidentifiable rotations of a set of points, the configuration. It is tacitly assumed that the set of points under consideration is materially uniform. Otherwise, a set of rotations would become identifiable due to just the arrangement of the set of points under consideration. Because material symmetry, in continuum mechanics, is the inherent material property of the point under consideration and not the arrangement or the nature of the neighboring points, the requirement of the material uniformity arises. Further, the framework of continuum mechanics allows one to have homogeneous body made of anisotropic material or inhomogeneous body made of isotropic material.

Alternatively for some, material symmetry is related to the materials internal structure. Material symmetry, in this point of view, is the set of rotations of the body that leave its internal structure unaltered. In this definition of material symmetry based on its internal structure, homogeneous body made of anisotropic material or inhomogeneous body made of isotropic material is not possible. Also, in this case, the material symmetry is not a concept associated with a material point in the body.

As articulated by Lekhnitskii (1981), it is necessary to distinguish the symmetry in the constitutive relation versus the symmetry of the material based on its internal structure. Lekhnitskii (1981) sights Neumann (1885) for the assumption that the symmetry in the constitutive relation to be not inferior to that of the symmetry in the crystallographic structure. That is the set of rotations that form the symmetry group for a given crystallographic structure should always be contained in the set of rotations for which the constitutive relation remains invariant. Lekhnitskii (1981) then states that this assumption is extended to bodies that are not made of crystals but still have an internal structure like wood, glass fiber reinforced plastics. However, to the knowledge of the authors there has been no experimental validation of this assumption of the symmetry in the constitutive relation be not inferior to the symmetry in the internal structure. Hence, here an hypothesis using which the symmetry requirement of the constitutive relation could be established is sought.

It is found that when a body is made of isotropic material and is deforming from its undistorted state in a non-dissipative manner, then the principal direction of the left Cauchy–Green deformation tensor does not change with the magnitude of the applied uniaxial load. Further, it is also inferred that if any of the principal directions of the stress tensor does not coincide with the fiber direction then the principal direction of the left Cauchy–Green deformation tensor changes with the magnitude of the applied uniaxial stress. Therefore, if the principal direction of the left Cauchy–Green deformation tensor does not change with the magnitude of the applied uniaxial load while the body is being subjected to a non-dissipative process, for two different directions of the applied uniaxial load which are not orthogonal to each other, then the tested material could be inferred as being isotropic.

Homogenization procedures used to generate homogeneous constitutive relations for inhomogeneous bodies made of isotropic materials result in anisotropic constitutive relations (Nemat-Nasser and Hori, 1993). Therefore, it would be of interest to examine the quality of approximating inhomogeneous bodies made of isotropic materials with homogeneous but anisotropic constitutive relations. This study shows that the way in which the principal direction of the left Cauchy–Green deformation tensor vary with the magnitude of the applied uniaxial load depends on the material symmetry of the constitutive relation. Thus, if the direction in which the maximum stress and/or change in length occurs could not be predicted by these homogeneous anisotropic models for inhomogeneous bodies made of isotropic material, the engineering usefulness of these homogenization procedures is limited.

This article is organized in four sections including this introduction. In Section 2 the notations and well established relationships are documented for further reference. Then, in Section 3, it is established that the principal direction of left Cauchy–Green deformation tensor measured in an inhomogeneous body made of isotropic material would not change with the magnitude of the applied uniaxial load as long as it deforms in a non-dissipative manner and the deformation is measured from a stress free reference configuration. On the other hand, a body made of transversely anisotropic material the principal direction of left Cauchy–Green deformation tensor does change with the magnitude of uniaxial stress, provided that any of the principal directions of the stress does not coincide with the fiber direction. Finally, issues in testing this hypothesis experimentally is discussed.

2. Preliminaries

Let $\mathbf{X}$ denote the position vector of a typical particle belonging to the reference configuration of the body. Similarly, let $\mathbf{x}$ denote the position vector of the same particle in the current configuration. The deformation field of the body is defined through a one to one mapping $\boldsymbol{\chi}$ that tells the current position of the particle