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General transmission conditions for thin elasto-plastic pressure-dependent interphase between dissimilar materials

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ABSTRACT

We consider a thin soft adhesive interphase between dissimilar elastic media. The material of the intermediate layer is modelled by elasto-plastic pressure-sensitive constitutive law. An asymptotic procedure, together with a novel formulation of the deformation theory of plasticity for pressure-sensitive materials, is used in order to derive nonlinear transmission conditions for the corresponding imperfect zero-thickness interface. A FEM analysis of the original three-phase structure is performed to validate the transmission conditions for the simplified bimaterial structure.

1. Introduction

Adhesive joints are widely used in structural engineering applications involving composite materials, such as marine (Golaz et al., 2013), aerospace (Bhowmik et al., 2009) and automotive (Loureiro et al., 2010) structures. In these structures, polymeric adhesives are used to join different materials such as plastics, metals, fibre reinforced composites and others.

Early models of adhesive joints tended to assume that the adhesive behaves as a linear-elastic solid (Bikerman, 1961; Adams and Peppiatt, 1974). However, many adhesives (e.g. rubber modified epoxies) exhibit large plastic strains before failure (Wang and Chalkley, 2000). A number of yield criteria have been proposed to model the plastic behaviour of polymers. Among these, the Tresca and von Mises criteria, originally developed to describe the yield behaviour of metals, have been also used. However, these criteria cannot accurately predict the behaviour of polymeric adhesive specimens under multiaxial loading, as yielding in these materials is sensitive to hydrostatic as well as deviatoric stress (Rabinowitz et al., 1970; Altenbach and Tushtev, 2001). As a consequence, pressure-dependent plasticity theory is more appropriate in this case. In this context, the Drucker–Prager yield criterion is widely used for polymeric materials.

The numerical FE modelling of thin adhesive interphases is complicated by the very large aspect ratio of the intermediate layer, which requires extremely refined meshes. Alternatively, the so-called imperfect interface approach can be adopted. It consists of replacing the actual thin interphase between the two adherents by a zero thickness imperfect interface. The interface is supplied with special transmission conditions, which incorporate information about geometrical and mechanical properties of the original thin interphase.

Although imperfect transmission conditions for elastic interphases have been intensively investigated (Benveniste, 1985; Benveniste and Miloh, 2001; Avila-Pozos et al., 1999; Hashin, 2002; Mishuris 1999; Mishuris, 2001; Mishuris and Öchsner, 2005a), analytical results for elastoplastic adhesive interphase are very scarce and mainly limited to rigid adherents and pressure-independent plasticity (Ikeda et al., 2000; Mishuris and Öchsner, 2005b).

In this paper, nonlinear transmission conditions for a soft elasto-plastic *pressure-dependent* interphase are derived by asymptotic techniques. The interphase material is described by a novel formulation of the deformation theory for pressure-dependent media. Early models of pressure-dependent deformation theory can be found in Durban and Papanastasiou (2003) and Mishuris et al. (2013). However, the model proposed by Durban and Papanastasiou (2003) only accounts for elastic and plastic branches separately, i.e. the yield condition is not incorporated into the generalised constitutive parameters. On the other hand, the model proposed in Mishuris et al. (2013) applies only to a few specific loading paths. To overcome these limitations, the classical Hencky deformation theory is generalised in Section 3 to the case of pressure-sensitive materials by including the dependence of inelastic deformation on the first stress invariant, in the spirit

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suggested by Chen and Han (1988) and Lubarda (2000). The general theory is then specialised to the case of Drucker–Prager model with associated flow rule and isotropic linear hardening. More general models for full elastoplastic response of pressure sensitive solids for large strain Drucker–Prager and Mohr–Coulomb non-associative flow and deformation analyses were presented by Durban and Papanastasiou (1997).

The transmission conditions are validated by numerical examples based on accurate finite element simulations. The accuracy and efficiency of the proposed approach prove to be very high for different monotonic loading conditions.

2. Problem formulation and transmission conditions for elasto-plastic pressure-dependent interphases

Consider a structure composed by two dissimilar elastic materials joined together by a thin adhesive interphase, see Fig. 1. The thickness of the interphase is small in comparison with the characteristic size of the body: $h = \epsilon h_* \ll L,\ h_* \sim L$. Here ϵ is a small positive parameter, $\epsilon \ll 1$.

The adhesive material is assumed to be soft in comparison with the two adherents and may exhibit a very general nonlinear constitutive behaviour, including compressible plastic deformations, the only assumption being that the material is isotropic.

Adopting the deformation theory of plasticity, the constitutive laws of the interphase in the plastic regime will be described in terms of nonlinear elasticity as

$$\sigma_{ij} = \tilde{\lambda}\varepsilon_{kk}\delta_{ij} + 2\tilde{\mu}\varepsilon_{ij}, \tag{1}$$

where the generalised Lamé parameters are functions depending on the deformation within the interphase

$$\tilde{\lambda} = \tilde{\lambda}(J_1^\varepsilon, J_2^e), \quad \tilde{\mu} = \tilde{\mu}(J_1^\varepsilon, J_2^e). \tag{2}$$

Note that, due to the isotropy assumption, the dependence is written in terms of the two invariants

$$J_1^\varepsilon = \varepsilon_{kk}, \quad J_2^e = \frac{1}{2}e_{ij}e_{ij}, \tag{3}$$

where e_{ij} denotes the deviatoric part of the strain tensor. For the sake of keeping the analysis simple, the dependence on the third invariant is omitted. However, if necessary, the model can be generalised to include the effect of all invariants.

In the elastic regime, the elastic constants are assumed to be constant

$$\lambda = \epsilon\lambda_0, \quad \mu = \epsilon\mu_0, \tag{4}$$

where the elastic parameters λ_0, μ_0 are comparable with those of the adherent materials, $\lambda_\pm,\ \mu_\pm$.

We assume in our analysis that, for any admissible deformation of the material within the interphase, there exists a constant parameter ν_* such that

$$0 \leqslant \tilde{\nu}(J_1^\varepsilon, J_2^e) \leqslant \nu_* < 1/2, \tag{5}$$

where $\tilde{\nu}(J_1^\varepsilon, J_2^e)$ is the generalised Poisson's ratio

$$\tilde{\nu}(J_1^\varepsilon, J_2^e) = \frac{\tilde{\lambda}(J_1^\varepsilon, J_2^e)}{2\left[\tilde{\lambda}(J_1^\varepsilon, J_2^e) + \tilde{\mu}(J_1^\varepsilon, J_2^e)\right]}. \tag{6}$$

Under such assumption, the following two imperfect transmission conditions linking the values from different parts of the interphase are asymptotically satisfied, see Mishuris et al. (2013)

$$[\![\boldsymbol{t}_2]\!](x_1, x_3) = 0 \tag{7}$$

and

$$\boldsymbol{t}_2(x_1, x_3) = \frac{1}{2h}\boldsymbol{A}(x_1, x_3)[\![\boldsymbol{u}]\!](x_1, x_3), \tag{8}$$

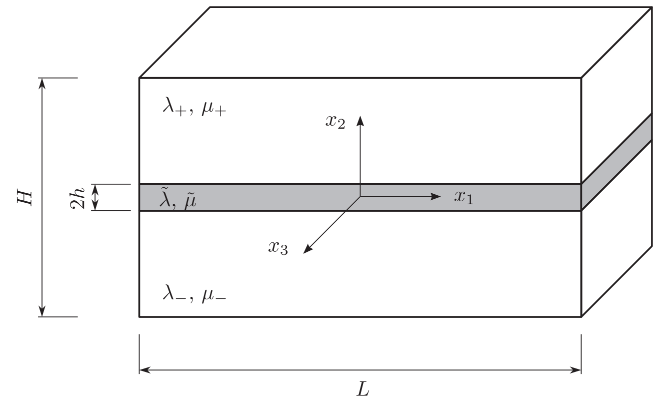

Fig. 1. Bimaterial structure with thin soft adhesive joint.

where $\boldsymbol{t}_2 = [\sigma_{21}, \sigma_{22}, \sigma_{23}]^T$ is the traction on the interphase surfaces, $\boldsymbol{u} = [u_1, u_2, u_3]^T$ is the displacement vector, and the jump of a function f across the interphase is denoted by $[\![f]\!] = f(x_2 = h) - f(x_2 = -h)$, and the tensor $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \begin{pmatrix} \tilde{\mu} & 0 & 0 \\ 0 & \tilde{\lambda} + 2\tilde{\mu} & 0 \\ 0 & 0 & \tilde{\mu} \end{pmatrix}. \tag{9}$$

The transmission conditions (7) and (8) are valid under the following ellipticity conditions

$$0 < \tilde{\mu} < \epsilon\mu_*, \quad 0 < \tilde{\lambda} + 2\tilde{\mu} < \epsilon\Lambda_*, \tag{10}$$

where μ_* and Λ_* are constants comparable in values with those of the adherent materials. In fact, we can take $\mu_* = \mu_0$ and $\Lambda_* = \lambda_0 + 2\mu_0$.

The asymptotic procedure allows us also to estimate the components of the strain tensor, which to the leading order $O(\epsilon^{-1})$ read[1]

$$\varepsilon_{ij} = O(1), \quad \varepsilon_{i2} = \frac{1}{2}\frac{\partial u_i}{\partial x_2} + O(1), \quad i, j = 1, 3, \quad \varepsilon_{22} = \frac{\partial u_2}{\partial x_2}. \tag{11}$$

By application of the generalised Hooke's law (1), we obtain the following relationships

$$\varepsilon_{i2} = \frac{\sigma_{i2}}{2\tilde{\mu}}, \quad i, j = 1, 3, \quad \varepsilon_{22} = \frac{\sigma_{22}}{\tilde{\lambda} + 2\tilde{\mu}}, \tag{12}$$

Finally, the leading terms of the strain tensor invariants can be calculated as

$$J_1^\varepsilon = \varepsilon_{22} = \frac{\sigma_{22}}{\tilde{\lambda} + 2\tilde{\mu}}, \tag{13}$$

$$J_2^e = \frac{1}{3}\varepsilon_{22}^2 + \varepsilon_{12}^2 + \varepsilon_{23}^2 = \frac{1}{3}\frac{\sigma_{22}^2}{(\tilde{\lambda} + 2\tilde{\mu})^2} + \frac{1}{4}\frac{\sigma_{12}^2}{\tilde{\mu}^2} + \frac{1}{4}\frac{\sigma_{23}^2}{\tilde{\mu}^2}. \tag{14}$$

Note that the material functions $\tilde{\lambda}$ and $\tilde{\mu}$, at this point, are not specified and should be defined depending on the particular elasto-plastic model adopted for the interphase material. However, all the stress components σ_{i2} of the traction vector $\boldsymbol{t}_2$ as well as the components of the strain tensor ε_{ij} in their leading terms are functions only of the space variables x_1 and x_3. This immediately implies that, in the case under consideration, the strain invariants are also functions of those two variables,

$$J_1^\varepsilon = J_1^\varepsilon(x_1, x_3), \quad J_2^e = J_2^e(x_1, x_3). \tag{15}$$

[1] The notation $f(\epsilon) = O(g(\epsilon)), \epsilon \to 0$ means that the function $f(\epsilon)$ is bounded above by the function $g(\epsilon)$, up to a constant factor, for ϵ sufficiently small.