International Journal of Solids and Structures 64–65 (2015) 62–70

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A note on stress fields and crack growth in porous materials subjected to a contact load

P. Isaksson

Solid Mechanics, The Ångström Laboratory, Uppsala University, Box 534, SE-751 21 Uppsala, Sweden

ARTICLE INFO

Article history:
Received 14 August 2014
Received in revised form 18 January 2015
Available online 8 April 2015

Keywords:
Porous material
Gradient enhanced continuum theory
Hertz contact

ABSTRACT

In materials where inherent heterogeneities like cells and pores are relatively large compared to other relevant mechanical dimensions, such as the contact surface in indentation tests or the length of an existing crack, stress and strain fields predicted by classical continuum elasticity theories may become too harsh because of absence of internal lengths in the equations that characterize the underlying microstructure. To overcome this deficiency a gradient enhanced elasticity continuum theory may be applied, which include length parameters in the constitutive equations that limit the magnitude of deformation gradients and is able to capture internal length effects. In this study, microscopic stress fields in porous materials subjected to a cylindrical contact load are estimated with such a gradient enhanced continuum model. To judge the model's ability to capture the mechanical behavior in this class of materials, calculated stress fields in the contact region, given by the gradient theory, are contrasted with microscopic stress fields computed in discrete high-resolution finite element models of cellular wood-like structures having varying average pore sizes but identical macroscopic geometry and boundary conditions as the gradient model. X-ray computed tomography experiments on wood illustrate the phenomenon. It is observed, in both experiment and finite element models, that the region of high shear stresses, where a crack may grow despite a confining pressure, is located deeper down in the material than what is predicted in classical continuum theories. On the other hand, the gradient enhanced model produces remarkably similar stress/strain fields as the finite element models and experiment and is thus seemingly able to capture microstructural size effects.

1. Introduction

Criteria for brittle fracture initiation and growth have been the subject of many studies in attempts to capture the essential features of the physical mechanisms leading to crack initiation and growth (cf. Bergqvist and Guex, 1979; Cotterell and Rice, 1980; He and Hutchinson, 1989). An overwhelming majority of the studies are focused on the mechanisms of opening (mode I) cracks. Even though a crack in some situations may grow in shear mode (mode II) under a pure shear load, they usually grow under mode I as a result of local tensile stresses at the tip, cf. Broberg (1999). However, if shear stresses are present in combination with a sufficiently high confining pressure, opening of the crack is prevented and shear mode crack growth occur in the direction of the principal shear stress, cf. Lawn and Wilshaw (1975), Melin (1986) or Isaksson and Ståhle (2001, 2002). One practical situation where such fractures are promoted is in indentation tests. Indentation testing, both on the macro and micro scales, is increasingly used to determine mechanical properties of materials and is conducted by pressing a stiff indenter to a plane surface. The applied load results in compressive and shear stresses in the material well below the contact surface. In theory, material parameters like Young's modulus, hardness and fracture toughness can be estimated (cf. Kruzic et al., 2009). Nowadays, not only traditional "continuum-like" materials such as metals, ceramics or PMMA are tested but also considerably more complex heterogeneous materials like composites or soft and hard tissues are evaluated (cf. Imbeni et al., 2005; Mullins et al., 2007). However, in materials where inherent heterogeneities like cells and pores are relatively large compared to the size of the contact surface, stress and strain fields predicted by classical continuum elasticity theories may become too harsh because of the absence of internal lengths in the equations that characterize the underlying microstructure. To overcome this deficiency, a nonlocal or gradient elasticity theory may be applied that include length parameters in the constitutive equations, cf. Askes and Aifantis (2011) or Isaksson and Dumont (2014). A mathematical framework to introduce gradient theories

E-mail address: per.isaksson@angstrom.uu.se

http://dx.doi.org/10.1016/j.ijsolstr.2015.03.014

was originally presented in the early 1960s (Toupin, 1962; Kröner, 1963). During that decade much work emerged in the field (Mindlin, 1965; Green and Rivlin, 1964; Kröner, 1967; Mindlin and Eshel, 1968). Since then have numerous theories incorporating material length scales been presented (cf. Eringen and Edelen, 1972; Triantafyllidis and Aifantis, 1986; Eringen, 2002; Aifantis, 2003; Isaksson and Hägglund, 2013). The simplicity and attraction of the pioneer Aifantis and co-workers' formulations relies in the fact that only one additional constitutive constant is required. However, as been observed by Elias Aifantis in a recent review article (Aifantis, 2014), there is still no available framework for relating internal lengths to physical properties even though the need has been documented in several studies (among others: Fleck and Hutchinson, 2001; Ghoniem and Walgraef, 2008; Gurtin and Anand, 2009). Internal lengths are usually treated as pure phenomenological constants and, with few exceptions, not much progress has been made toward their physical identification through microstructure modeling simulations or laboratory tests. The present study is an attempt towards trying to meet this need and the aim here is to analyze micro stress field distributions in porous materials subjected to a contact load using a special implicit enhanced elasticity gradient theory derived from a nonlocal theory. Special attention is given to the interaction of different length scales, i.e. the size of the contact surface and the microstructural pore size. The special case of a classical Hertz cylindrical contact is investigated, but the model can be applied to any contact geometry having a reasonably correct approximated pressure distribution. To judge the enhanced gradient continuum model, stress fields in the region of the contact are contrasted with stress fields computed in discrete high-resolution finite element structural models having different microstructural pore sizes but identical macroscopic geometry and boundary conditions as the gradient continuum model.

2. Experimental observations

Wood is analyzed in X-ray computed tomography experiments to illustrate the mechanical behavior when a hard tip is pressed to a porous material. The equipment used, a SkyScan-1172, has a resolution high enough to distinguish individual cells. The instrument has a built-in compression stage, which was used to press a small indenter to the wood's surface. The indenter, made of steel, was needle shaped with a 200 μm radius spherical tip. The specimen was mounted on a slowly rotating holder that allows X-rays to enter from different directions. In this way, two-dimensional projection images were taken in a multitude of directions, which allows subsequent reconstruction of a 3D microstructure. The loading was stepwise increased allowing for successive 3D images showing the complete deformation process when the indenter loads the surface. During each scan, 998 radiographs were taken over 180 degrees with a pixel size of 2.44 μm. The cross-section analyzed was the radial-tangential plane, i.e. a plane of the cross-section of the cells, Fig. 1.

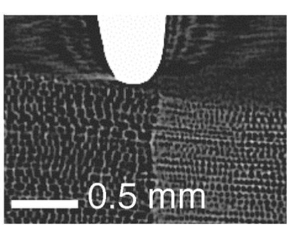

Fig. 1. Reconstructed cross-section image of the wood specimen. The indenter is the white object close to the wood surface. The average cell diameter in the region below the tip is approximately ~65 μm.

Then, comparing cross-section images at two consecutive load steps, 2D deformation fields of the wood's internal structure were calculated by a digital image correlation algorithm (Blaber et al. 2015). Two consecutive images were picked immediately before and after contact between the indenter and surface, meaning that the stress state in the wood most likely remained in a linear elastic regime while keeping the influence from nonlinear deformations caused by plasticity or damage at such a low level as possible. For most materials, a random or regular pattern must be applied to a surface of the specimen, which deforms along with the object, when performing these types of image analyses. However, for wood materials there is no need for this due to the inherent pattern in the wood's microstructure. The random variations in density and thickness of cell walls are a sufficient reference pattern. The deformation of an approximately 2×2 mm domain in the indented wood was used for the calculations. Regions having high shear strains – where mode II fractures most likely initiates – are found to be in a fairly large region below the tip. Figs. 2(a) and (b) shows contours of the estimated principal shear strain field and hydrostatic stress, given by the experimentally estimated normal strain components (for simplicity, an isotropic material behavior is assumed). Clearly, both shear and compressive stresses are present in the region below the tip. The experimental shear strain field in Fig. 2(a) should be contrasted to the theoretical shear strains in Fig. 3, given by the classical Hertz elasticity theory (cf. Johnson, 1985).

The Hertz theory will be further discussed in Section 3. Meanwhile, one may observe in Figs. 2 and 3 that the experimentally estimated strain field is considerably smoother than the theoretical strain field in which higher gradients are present. Also, the region of relatively high experimental shear strains is larger, as compared to the theoretical field, and located almost three times deeper down in the material. Thus, the strain field in the porous material is somewhat different than in an ideal elastic homogeneous continuum. This experimental observation indicates that the material's complex

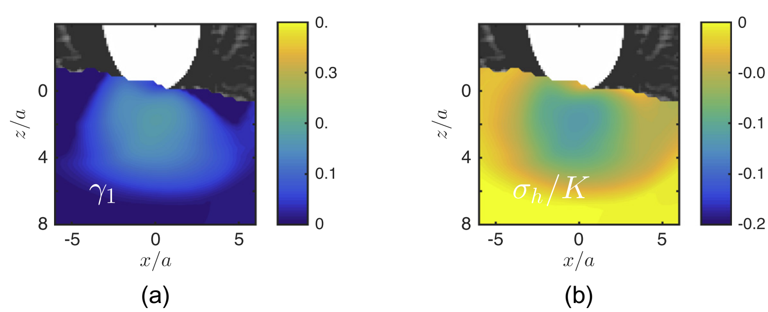

Fig. 2. (a) Experimentally estimated principal shear strain γ_1 and (b) experimentally estimated hydrostatic stress $\sigma_h/K = \varepsilon_{xx} + \varepsilon_{zz} + \varepsilon_{yy}$ (K is the bulk modulus while ε_{xx}, ε_{zz} and ε_{yy} are the experimentally estimated normal strains). The spatial positions are normalized with a, where $2a \approx R$ is the contact width.