International Journal of Solids and Structures 64–65 (2015) 71–85

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

In-plane stress analysis of cracked layers under harmonic loads

A.M. Baghestani [a], S.J. Fariborz [a,*], A.R. Fotuhi [b]

[a] Department of Mechanical Engineering, Amirkabir University of Technology (Tehran Polytechnic), 424, Hafez Avenue, Tehran, Iran
[b] Department of Mechanical Engineering, Yazd University, Safaeieh, Yazd, Iran

ARTICLE INFO

Article history:
Received 17 September 2014
Received in revised form 16 March 2015
Available online 27 March 2015

Keywords:
In-plane
Isotropic layer
Harmonic load
Edge dislocation
Multiple cracks

ABSTRACT

Stress analysis is carried out in an isotropic layer weakened by multiple cracks under time-harmonic loading. The viscous damping is used to model energy dissipation in the material. The analysis is based on the stress fields caused by climb and glide of an edge dislocation in the layer. The solution for dislocation is obtained by means of the integral transform method. Furthermore, stress analysis in the intact layer under self-equilibrating harmonic point loads is carried out. These solutions are employed to derive a set of Cauchy singular integral equations for analyzing cracks parallel/perpendicular to the layer boundary. The numerical solution of these equations yields dislocation densities on a crack surface which are used to determine dynamic stress intensity factors over a range of frequency. The natural frequencies of layers with horizontal cracks are obtained and the interaction of multiple cracks is studied.

1. Introduction

Structures under dynamic loads are vulnerable to fatigue failure. An accurate stress analysis is of paramount importance for a reliable life estimation of a mechanical component under periodic loads. In particular, in bodies with multiple cracks subject to dynamic loads, owing to the formation of regions with high stress gradient and also interaction among cracks, stress evaluation is a complicated problem. For cracked regions with simple geometries, however, analytical procedures may be employed to achieve the task. The solutions may be used to benchmark the results obtained from numerical procedures. A brief review of literature concerning in-plane periodic excitation of cracked media is presented here. Mal (1970) studied the diffraction of normally incident longitudinal and anti-plane shear waves by a Griffith crack in an isotropic infinite plane. The problem of diffraction of stress wave by a crack perpendicular to the interface of two dissimilar half-planes was considered by Loung et al. (1975). A set of singular integral equations was derived by means of integral transform technique. These equations were solved numerically and the effects of material properties and crack distance from the interface on the stress intensity factors (SIFs) were analyzed. A half-plane weakened by a subsurface crack perpendicular to its boundary was solved by Achenbach and Brind (1981). The half-plane was under time-harmonic tractions and the Fourier transform was employed to obtain modes *I* and *II* SIFs. The variation of SIFs versus load frequency for different locations of crack was investigated. The mode *I* analysis of a Griffith crack which was perpendicular to the boundaries of an isotropic strip and excited by a longitudinal harmonic load was accomplished by Srivastava et al. (1981). Keer et al. (1984) analyzed a subsurface crack parallel to the boundary of a half-plane under time-harmonic excitation. The problem formulated as a system of integral equations for the dislocation densities. These equations were solved numerically and SIFs were obtained. The resonance phenomenon was observed as crack approached the free surface of half-plane. Qu (1994) studied the stress fields caused by a finite crack along the interface of two dissimilar half-planes under harmonic elastic waves. The behavior of crack-tip singular fields was oscillatory with the index equal to that of cracks under static loading. The interaction between an arbitrarily oriented micro defect and a main crack subjected to a plane incident wave was studied by Meguid and Wang (1995). The effects of wave frequency and micro crack orientation on the SIFs of the main crack were also analyzed. Itou (1996) considered an infinite orthotropic plane weakened by two collinear cracks under normal harmonic elastic wave. The problem was reduced to a set of dual integral equations which were solved by means of Schmidt method. An article by Zhou et al. (2004) deals with a finite crack in a functionally graded plane under time harmonic loading. The material properties of the plane, Young's modulus and mass density, were assumed to vary exponentially perpendicular to the direction of the crack surface while Poisson's ratio was constant. The integral transform and Schmidt methods were utilized for the solution and the effect of excitation frequency on SIF was investigated. Ayatollahi and Fariborz (2009) solved the

* Corresponding author. Tel.: +98 2164543460; fax: +98 216419736.
E-mail address: sjfariborz@yahoo.com (S.J. Fariborz).

http://dx.doi.org/10.1016/j.ijsolstr.2015.03.015
0020-7683/

problem of multiple smooth cracks in an infinite isotropic plane under time-harmonic loading. They used distributed dislocation technique to construct a set of Cauchy singular integral equations for the problem. These equations were solved numerically to analyze interaction between cracks.

All the above investigations were either for infinite planes with cracks or layers weakened by a single crack. In this article, the problem of climb and glide of an edge dislocation in a layer subjected to time-harmonic loading is solved. The dislocation cut is either parallel or perpendicular to the layer boundaries. The structural energy dissipation is taken into account and it is modeled by viscous damping. By means of the distributed dislocation technique, Hills et al. (1996), the dislocation solutions are used to formulate integral equations for a layer weakened by multiple horizontal/vertical embedded cracks. Contrary to the static case, however, the formulation may not be used for the analysis of oblique cracks. Moreover, crack closing is not permitted. Therefore, in addition to dynamic load a sufficiently large static force, to prevent cracks closing, may be applied on the layer. However, the solution to static case is given in Fotuhi and Fariborz (2008); thus, by virtue of superposition principle, it suffices to analyze the dynamic problem. For a small value of load frequency the static results are recovered. The first critical frequency of cracked layers is obtained and the effects of damping on stress intensity factors of cracks are investigated. Furthermore, the interaction between cracks is studied. In general, the time-harmonic solution may be used, via Fourier synthesis, for the steady-state analysis of a problem.

2. Dislocation solution

In the plane infinitesimal theory of elasticity, the constitutive equations for isotropic materials are expressed as

$$\begin{aligned}\sigma_{XX} &= \frac{\mu}{\kappa-1}\left[(\kappa+1)\frac{\partial U_X}{\partial X}+(3-\kappa)\frac{\partial U_Y}{\partial Y}\right]\\ \sigma_{YY} &= \frac{\mu}{\kappa-1}\left[(\kappa+1)\frac{\partial U_Y}{\partial Y}+(3-\kappa)\frac{\partial U_X}{\partial X}\right]\\ \sigma_{XY} &= \mu\left(\frac{\partial U_X}{\partial Y}+\frac{\partial U_Y}{\partial X}\right)\end{aligned} \tag{1}$$

where μ is the shear modulus of elasticity, κ is the Kolosov constant of the medium, U_X and U_Y are displacement components in the X and Y directions, respectively. The Kolosov constant is $\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3-\nu)/(1+\nu)$ for plane stress situations, where ν is the Poisson's ratio. We assume that energy dissipation in the body which may be attributed to the formation of plastic region around crack tips is proportional to the velocity of motion, i.e., viscous damping. Thus, equations of motion yield

$$\begin{aligned}\frac{\partial\sigma_{XX}}{\partial X}+\frac{\partial\sigma_{XY}}{\partial Y} &= \rho\frac{\partial^2 U_X}{\partial t^2}+\eta\frac{\partial U_X}{\partial t}\\ \frac{\partial\sigma_{YY}}{\partial Y}+\frac{\partial\sigma_{XY}}{\partial X} &= \rho\frac{\partial^2 U_Y}{\partial t^2}+\eta\frac{\partial U_Y}{\partial t}\end{aligned} \tag{2}$$

where ρ is the mass density and η is the damping coefficient of material. From Eqs. (1) and (2), we arrive at Navier's equations as

$$\begin{aligned}\mu\left(\frac{\partial^2 U_X}{\partial X^2}+\frac{\partial^2 U_X}{\partial Y^2}\right)+\frac{2\mu}{\kappa-1}\frac{\partial}{\partial X}\left(\frac{\partial U_X}{\partial X}+\frac{\partial U_Y}{\partial Y}\right) &= \rho\frac{\partial^2 U_X}{\partial t^2}+\eta\frac{\partial U_X}{\partial t}\\ \mu\left(\frac{\partial^2 U_Y}{\partial X^2}+\frac{\partial^2 U_Y}{\partial Y^2}\right)+\frac{2\mu}{\kappa-1}\frac{\partial}{\partial Y}\left(\frac{\partial U_X}{\partial X}+\frac{\partial U_Y}{\partial Y}\right) &= \rho\frac{\partial^2 U_Y}{\partial t^2}+\eta\frac{\partial U_Y}{\partial t}\end{aligned} \tag{3}$$

In a time-harmonic motion with angular frequency ω, the time dependency of displacement field is expressed as

$$[U_X(X,Y,t), U_Y(X,Y,t)] = [u(X,Y), v(X,Y)]e^{i\omega t} \tag{4}$$

where $i = \sqrt{-1}$. Eq. (3) in view of (4) reduces to

$$\begin{aligned}\frac{\partial^2 u}{\partial X^2}+\frac{\partial^2 u}{\partial Y^2}+\frac{2}{\kappa-1}\frac{\partial}{\partial X}\left(\frac{\partial u}{\partial X}+\frac{\partial v}{\partial Y}\right)+\omega\frac{\rho\omega-i\eta}{\mu}u &= 0\\ \frac{\partial^2 v}{\partial X^2}+\frac{\partial^2 v}{\partial Y^2}+\frac{2}{\kappa-1}\frac{\partial}{\partial Y}\left(\frac{\partial u}{\partial X}+\frac{\partial v}{\partial Y}\right)+\omega\frac{\rho\omega-i\eta}{\mu}v &= 0\end{aligned} \tag{5}$$

Henceforth, the factor $e^{i\omega t}$ common to all field variables is omitted. It is worth mentioning that, in practice energy dissipation depends on frequency of motion. The present analysis is capable of handling this dependency. We consider a layer with thickness h containing a Volterra edge dislocation with Burgers vectors b_X and b_Y, associated with climb and glide of the dislocation, respectively. The dislocation is situated at $(0,\xi)$, Fig. 1, and the dislocation cut is perpendicular to the boundary of the layer. The dislocation may be identified as

$$\begin{aligned}u(0^+,Y)-u(0^-,Y) &= b_X H(Y-\xi)\\ v(0^+,Y)-v(0^-,Y) &= b_Y H(Y-\xi)\end{aligned} \tag{6}$$

where $H(.)$ is the Heaviside step function. Moreover, the continuity of traction on the dislocation cut requires that

$$\begin{aligned}\sigma_{XX}(0^-,Y) &= \sigma_{XX}(0^+,Y)\\ \sigma_{XY}(0^-,Y) &= \sigma_{XY}(0^+,Y), \quad Y>\xi\end{aligned} \tag{7}$$

It is expedient to decompose the problem into symmetric and anti-symmetric problems with respect to the Y-axis and consider the half layer $X > 0$. In the symmetric problem, we deduce from conditions (6) and (7) that

$$u(0,Y) = \frac{b_X}{2}H(Y-\xi), \quad \sigma_{XY}(0,Y) = 0 \tag{8}$$

whereas, in the anti-symmetric case, the conditions become

$$v(0,Y) = \frac{b_Y}{2}H(Y-\xi), \quad \sigma_{XX}(0,Y) = 0 \tag{9}$$

Moreover, the traction free condition on the layer surface implies that

$$\begin{aligned}\sigma_{YY}(X,0) &= 0, \quad \sigma_{YY}(X,h) = 0\\ \sigma_{XY}(X,0) &= 0, \quad \sigma_{XY}(X,h) = 0\end{aligned} \tag{10}$$

In the former problem, we eliminate variable X by applying Fourier sine and cosine transforms to first and second Eq. (5), respectively. Taking into account Eq. (8) and assuming that stress fields decay sufficiently rapidly as $X\to\infty$, results in

$$\begin{aligned}\frac{d^2u_s}{dY^2}-\frac{2\lambda}{\kappa-1}\frac{dv_c}{dY}-\left(\lambda^2\frac{\kappa+1}{\kappa-1}-\omega\frac{\rho\omega-i\eta}{\mu}\right)u_s &= -\lambda\frac{\kappa+1}{\kappa-1}\frac{b_X}{2}H(Y-\xi)\\ \frac{\kappa+1}{\kappa-1}\frac{d^2v_c}{dY^2}+\frac{2\lambda}{\kappa-1}\frac{du_s}{dY}-\left(\lambda^2-\omega\frac{\rho\omega-i\eta}{\mu}\right)v_c &= \frac{3-\kappa}{\kappa-1}\frac{b_X}{2}\delta(Y-\xi)\end{aligned} \tag{11}$$

where subscripts s and c designate, respectively, Fourier sine and cosine transforms of the relevant quantity, λ is Fourier transform variable and $\delta(.)$ is the Dirac delta function. Analogously, in the anti-symmetric problem, we apply Fourier cosine and sine transforms, respectively, to the first and second Eq. (5) and utilize conditions (9) to arrive at

$$\begin{aligned}\frac{d^2u_c}{dY^2}+\frac{2\lambda}{\kappa-1}\frac{dv_s}{dY}-\left(\lambda^2\frac{\kappa+1}{\kappa-1}-\omega\frac{\rho\omega-i\eta}{\mu}\right)u_c &= \frac{b_Y}{2}\delta(Y-\xi)\\ \frac{\kappa+1}{\kappa-1}\frac{d^2v_s}{dY^2}-\frac{2\lambda}{\kappa-1}\frac{du_c}{dY}-\left(\lambda^2-\omega\frac{\rho\omega-i\eta}{\mu}\right)v_s &= -\lambda\frac{b_Y}{2}H(Y-\xi)\end{aligned} \tag{12}$$