International Journal of Solids and Structures 64–65 (2015) 132–144

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Evaluation of material properties of incompressible hyperelastic materials based on instrumented indentation of an equal-biaxial prestretched substrate

Th. Zisis [a], V.I. Zafiropoulou [b], A.E. Giannakopoulos [b,*]

[a] Mechanics Division, National Technical University of Athens, 5 Heroes of Polytechnion Ave., Zographou GR-15773, Greece
[b] Laboratory of Strength of Materials and Micromechanics, Department of Civil Engineering, University of Thessaly, Volos 38336, Greece

ARTICLE INFO

Article history:
Received 21 November 2014
Received in revised form 9 March 2015
Available online 31 March 2015

Keywords:
Instrumented indentation
Hyperelasticity
Prestretching
Property extraction
Indentation strength

ABSTRACT

It is well known that instrumented indentation tests are useful tools in probing mechanical properties of materials such as metals and ceramics. Instrumented indentation of hyperelastic materials such as rubbers, bio-materials, tissues etc has not been examined in depth, especially the inverse problem of material characterization from instrumented indentation response. The difficulty of the inverse problem for such materials is that the unknown property is a function, the elastic energy density function. There are several such functions and each function is often characterized by more than one parameter. If the maximum indentation depth is low, we have shown that instrumented indentation of initially unstretched hyperelastic materials can only resolve a combination of the material parameters. If the maximum indentation depth is high, the indentation can provide independent material properties, however not in a unique way. Moreover, high indentation loads could lead to surface puncturing and so blur the test results. In this work, we show that we can use spherical indentation of a substrate at different but known prestretch levels to obtain the involved material properties of the energy density function. The present methodology can also incorporate a limit energy failure criterion and instrumented indentation can incorporate this behavior which we may call indentation strength.

1. Introduction

In recent years, instrumented indentation has been used to estimate residual stresses and residual strains of elastoplastic materials. Important analysis has been conducted for equal-biaxial stress fields in the context of sharp indentation tests (see for example Bolshakov et al. (1996), Suresh and Giannakopoulos (1998)) and for general biaxial stress fields (see for example Giannakopoulos (2003), Lee and Kwon (2004), Bocciarelli and Maier (2007), Larsson and Blanchard (2012)). Important studies were also conducted for the influence of residual stresses to spherical indentation (see for example Swadener et al. (2001) and Huber and Heerens (2008)). We should point out that this prior work is focused on how to obtain residual stresses and strains from instrumented indentation. The present work suggests a reverse methodology: obtaining material properties with the help of residual strains. Although we will concentrate mainly on hyperelastic materials, the present ideas could be useful for the elastoplastic materials, breaking the often encountered "lack of uniqueness" of elastoplastic material constants (see for example Chen et al., 2007).

Non linear finite element analysis was applied by Chang and Sun (1991) to (frictionless) spherical indentation of an Ogden type hyperelastic half-space. The general form of the Ogden's strain energy function is

$$W(\lambda_1,\lambda_2,\lambda_3)=\sum_{k=1}^{m}\mu_k(\lambda_1^{a_k}+\lambda_2^{a_k}+\lambda_3^{a_k}-3)/a_k \quad (1)$$

where λ_i $(i=1,2,3)$ define the principal values of the stretch tensor and μ_k, a_k are material constants. In the case of infinitesimal strains, the Ogden material behaves as an incompressible linear material with shear modulus G (and elastic modulus E)

$$G=\frac{1}{2}\sum_{k=1}^{m}\mu_k a_k=\frac{E}{3} \quad (2)$$

The neo Hookean material model is a special case of one-term Ogden model $(m=1, a_1=2)$. The Mooney–Rivlin material is a two-term Ogden model $(m=2, a_k=\pm 2, \mu_k=\pm C_k, k=1,2)$.

* Corresponding author. Tel.: +30 24210 74179.
E-mail address: agiannak@civ.uth.gr (A.E. Giannakopoulos).

http://dx.doi.org/10.1016/j.ijsolstr.2015.03.019
0020-7683/

Chang and Sun (1991) found that for a ratio of indentation depth/ball radius (h/R) up to 0.29, the load-indentation ($P-h$) depth relation is approximated well by the classic Hertz solution:

$$h = \left(\frac{9}{16}\frac{P^2}{E^2R}\right)^{1/3} \tag{3}$$

This $P-h$ result was found to be independent of other material parameters. Similar results have been found for the case of a flat punch and the cone punch for an extensive variety of strain energy functions (Giannakopoulos and Panagiotopoulos (2009), Zisis et al. (2011)). It is of interest to note that this lack of uniqueness of the $P-h$ curve has been also observed in the analysis of elastoplastic indentation by sharp indentors. In a most recent and extensive work on this subject, Chen et al. (2007) came up with a whole class of elastoplastic materials that show the same $P-h$ curves.

Spherical indentation of soft matter, beyond the Hertzian regime has been investigated numerically using hyperelastic models by Lin et al. (2009). They found that Hertz solution applies for $h/R < 0.2$, with the relation between the contact radius a, the ball radius R and the indentation depth h as

$$h = a^2/R \tag{4}$$

For $h/R > 0.2$, their results do not follow Hertz solution. In the context of linear elasticity, the solution of Ting (1966) which takes into account the exact spherical geometry gives:

$$h = \frac{1}{2}a\ln\frac{R+a}{R-a} \tag{5}$$

$$\frac{P}{2G} = (R^2+a^2)\ln\frac{R+a}{R-a} - 2aR \tag{6}$$

Clearly the above results approach Hertz solution for $h/R \ll 1$. However, finite element results performed by Lin and Chen (2006) for neo-Hookean material indicate that Ting's solution overestimates the actual load and displacement and the absolute error of Ting's solution is larger than that of the Hertz theory.

Green et al. (1952) have investigated the flat punch problem of an initially isotropic hyperelastic substance under equal-biaxial finite deformation. Woo and Shield (1962) investigated the semi-infinite media in biaxial extension with a superposed force normal to the boundary, using the methodology and the results of Harding and Sneddon (1945) and Elliot (1948). Equibiaxially stressed neo-Hookean half-space indented by a punch of arbitrary axisymmetric profile has been investigated by Dhaliwal et al. (1980) and for general hyperelastic (isotropic) bodies by Beatty and Usmani (1975).

Karduna et al. (1997) performed finite element analysis of a Mooney–Rivlin type of rubber under equal-biaxial stretching and confirmed the theoretical predictions of Humphrey et al. (1991). This approach has been used to investigate the constitutive behavior of living cells with atomic force microscopy, Na et al. (2004).

Dhaliwal and Singh (1978) presented analytical expressions of the incremental stress and displacement fields for the axisymmetrical indentation of initially stressed, in incompressible neo-Hookean solids, using the incremental theory of Biot (1965) and the potential theory. The results can be extended for the indentation by any smooth axisymmetric indenter, as verified by Yang (2004). The $h-a$ relation is given by the linear elastic solution and the $P-h$ relation admits an overall correction that depends on the amount of the initial biaxial stretch λ_r. For a neo-Hookean material, the finite tensile stretching makes it harder to deform under indentation.

The frictionless adhesion problem for a neo-Hookean material has been investigated numerically by Lin and Chen (2006) and they found that for $h/R < 0.5$, the classic solution of Johnson et al. (1971) holds well (the well known JKR model). The effect of finite stretching on adhesive contact between an axisymmetric indenter and a neo-Hookean solid with frictionless contact conditions was investigated by Yang (2011). It was found that the stretching has little effect on the surface interaction of solid surfaces while it changes the adhesive contact between the indenter and the surface. For neo-Hookean solid indented by a spherical indenter, the pull-off force (at $h=0$) is independent of the stretching as Johnson et al. (1971) predict. For this case, if γ is the interfacial energy density, Yang (2011) reports the results

$$P = \frac{16Ga^3}{3R}x \pm a(32\pi G\gamma a x)^{1/2} \tag{7}$$

$$a^3 = \frac{3P}{16Gx}\left[P + 3\pi\gamma R \pm \left(3\pi\gamma R^2 + 6\pi\gamma PR\right)^{1/2}\right] \tag{8}$$

$$x = \frac{-4\lambda_r^3 + (1+\lambda_r^6)^2}{2(\lambda_r^6-1)\lambda_r^4} \tag{9}$$

where λ_r is the initial biaxial stretch.

Of particular importance is the indentation of a thin layer of hyperelastic material that rests on a rigid substance under complete bonded or sliding condition. Such configurations are often encountered in actual testing conditions and the thickness t of the layer, as well as the bonding condition should be taken under considerations.

Regarding spherical indentation of a linear, isotropic and incompressible elastic material, Dimitriadis et al. (2002) provided an approximation where the Hertz solution should be modified by replacing the indentation depth with an equivalent indentation depth h_t according to:

$$\begin{aligned} \frac{h}{h_t} &= 1 + 0.884\left(\frac{a_t}{t}\right) + 0.781\left(\frac{a_t}{t}\right)^2 + 0.386\left(\frac{a_t}{t}\right)^3 \\ &\quad + 0.0048\left(\frac{a_t}{t}\right)^4 \quad \text{(non-bonded)} \\ \frac{h}{h_t} &= 1 + 1.133\left(\frac{a_t}{t}\right) + 1.283\left(\frac{a_t}{t}\right)^2 + 0.769\left(\frac{a_t}{t}\right)^3 \\ &\quad + 0.0975\left(\frac{a_t}{t}\right)^4 \quad \text{(bonded)} \end{aligned} \tag{10}$$

where the actual contact area is $a_t = \sqrt{Rh_t}$ (the load P is kept the same with that of $t \to \infty$).

The axisymmetric contact for an initially (equal-biaxial) stressed neo-Hookean elastic layers has been investigated by Dhaliwal et al. (1980), who presented numerical results for the flat punch and cone indenters that punch a bonded layer on a rigid substrate. Chen and Diebels (2012) indicate that the Hertz solution dominates for $h_t/t < 0.05$.

It is clear that extracting hyperelastic material properties from indentation tests is very complex. It fails to provide reliable and unique results, unless for high indentation depths. However, high indentation depth can cause damage of the material, especially for soft tissues (e.g. abdominal aortic aneurysms). In this work we will explore the indentation method in combination with pretressing, in order to extract hyperelastic material properties. Moreover, we will investigate the effect of substrate thickness, since most tests take place on thin specimens laying on rigid substrates. Finally we will include a local failure criterion in the form of a critical strain energy, as initially suggested by Volokh (2007).