International Journal of Solids and Structures 64–65 (2015) 246–258

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Macroscopic simulation of membrane wrinkling for various loading cases

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ARTICLE INFO

Article history:
Received 9 May 2014
Received in revised form 4 March 2015
Available online 9 April 2015

Keywords:
Fourier series
Membrane
Instability
Wrinkling
Asymptotic Numerical Method

ABSTRACT

The paper discusses membrane instability phenomena by a Fourier-related double scale approach that was introduced recently. This leads to a reduced-order model that is able to capture the main features of the wrinkles with few degrees of freedom. The paper focuses on the corresponding finite element procedure, its implementation and evaluation and applications to various cases of loading and boundary condition. The finite element model has first been implemented in a home-made code, the nonlinear system being solved by the Asymptotic Numerical Method (ANM), which has advantages of efficiency and reliability for stability analyses. It has also been implemented as a user element in a commercial software to evaluate the effectiveness of the reduction technique. Various loading cases were considered and the numerical tests establish that the reduced model can predict the wrinkling patterns, even when there are few wrinkles. The numerical results highlight the strong influence of a dimensionless parameter for wrinkling initiation.

1. Introduction

Membranes have recently seen a surge of interest in spacecraft structures (sunshields, solar sails, solar panels, reflector antennas and inflatable membranes), civil structures (pneumatic structures for roofs), biomedical materials. Due to the almost negligible bending stiffness and the associated inability to withstand compressive loading, wrinkles arise in very thin membranes, which has an adverse effect on the static and dynamic characteristics and longevity of membrane structures. Hence, to predict and avoid wrinkles, modeling and numerical analysis of membrane wrinkling has been a subject of interest.

The pioneer work on membrane wrinkling belongs to Wagner (1929) who developed a tension field theory, which assumes that out-of-plane flexural and in-plane compressive stress are negligible and bending stiffness is neglected. On the basis of tension field theory, many approaches have been put forward to address issues in membrane wrinkling. In most of them, see Stein and Hedgepeth (1961), Liu et al. (2001), Rossi et al. (2005), Jarasjarungkiat et al. (2008), the stress–strain relationship is modified to eliminate compressive stresses and a wrinkling criterion is established by distinguishing three membrane states: taut, wrinkled and slack. Another method by Roddeman et al. (1987,) splits the deformation tensor into two parts, the membrane part and wrinkling part. It has also attracted many researchers, see for instance Miyazaki (2006), Akita et al. (2007), Shaw and Roy (2007), Pimprikar et al. (2010). As the partial differential equations deduced from these membrane models are not elliptic (or hyperbolic in the dynamical case), the presence of slack regions may result in near singular stiffness matrices leading to difficulties in numerical solution. This problem can become well posed if an internal length is included, for instance within Cosserat theory (Banerjee et al., 2009; Pimprikar et al., 2010), but perhaps this regularization is not necessary when using an explicit dynamic computation. However, all these membrane models just characterize the stress distribution and the wrinkled region, but cannot identify the details of wrinkles such as the amplitude of the wrinkles, their wavelength, the sensitivity to boundary conditions and the instability critical load.

In general, shell elements accounting for very small bending stiffness are preferred to qualitatively characterize the details of membrane wrinkles. In recent papers, Wong and Pellegrino (2006) presented a general procedure for simulating the onset and growth of wrinkles with shell element using the commercial finite element package ABAQUS and obtained consistent results with experiments in Wong and Pellegrino (2006,). Wang et al.

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http://dx.doi.org/10.1016/j.ijsolstr.2015.04.003
0020-7683/

(2009) proposed a new Modified Displacement Component (MDC) method to eliminate the singularity of stiffness matrix and apply it to a finite element code ANSYS. Flores and Oñate (2011) used a rotation-free linear strain shell triangle element within explicit time integration strategy that avoids introducing geometric imperfection and initial stress. Lecieux and Bouzidi (2012) adopted the same shell element as Flores to analyze the wrinkling of membranes by direct energy minimization. In all these models, a large number of degrees of freedom are needed to capture the details of wrinkles that results in heavy computational cost and convergence is difficult to achieve. This kind of shell models will be referred as full shell model.

In this paper, we present a new finite element based on a reduced-order model established recently from a multiple scale analysis (Damil et al., 2013; Damil et al., 2014). It couples a nonlinear 2D membrane model with an envelope equation governing the evolution of wrinkles. This membrane theory is an extension of the Landau–Ginzburg bifurcation equation and it has been deduced by using the method of Fourier series with slowly variable coefficients (Damil and Potier-Ferry, 2010). The origin of the reduction lies in the double scale approach so that the needed spatial meshes can be considered as macroscopic and are not related with the wrinkling wavelength. The finite element technology is described in details and this has been implemented in a homemade code and also in the commercial package ABAQUS as a user element. In the first code, the governing equations are solved by the Asymptotic Numerical Method that is an efficient and robust path following technique in the presence of bifurcations. Several numerical simulations will be discussed in order to evaluate the range of validity of the reduced model, to establish the efficiency of the reduction and to analyze the case of a uniaxially stretched plate (Friedl et al., 2000; Jacques and Potier-Ferry, 2005) that is very sensitive to boundary conditions.

This paper is structured as follows. Section 2 recalls the famous Föppl–Von Karman equations for isotropic plates that will be considered as the reference model. In Section 3, the finite element procedure will be described. It includes the reduced-order model of Damil et al. (2013, 2014), a 2D finite element discretization and the resolution technique by ANM. In Section 4, two numerical examples are investigated to evaluate the range of validity and to bring out the strong influence of boundary conditions on wrinkling. In Section 5, a dimensionless parameter K is introduced that is important to predict the appearance of membrane instability. Finally, in Section 6, we describe the implementation into ABAQUS as user element (UEL), especially to obtain a reliable evaluation in terms of computation time.

2. Starting plate model

In this paper, the well known Föppl–Von Karman equations for elastic isotropic plates will be considered as the reference model. Sometimes we also use a commercial code, where the shell finite element model remains valid for large rotations, but the difference between these two models is weak in the cases considered in this paper. The Föppl–Von Karman equations can be written as:

$$\begin{cases} \mathrm{div}\mathbf{N} = 0, & (a) \\ \mathbf{N} = \mathbf{L^m} \cdot \boldsymbol{\gamma}, & (b) \\ 2\boldsymbol{\gamma} = \nabla\mathbf{u} + {}^t\nabla\mathbf{u} + \nabla w \otimes \nabla w, & (c) \\ D\Delta^2 w - \mathrm{div}(\mathbf{N}\cdot\nabla w) = 0. & (d) \end{cases} \tag{1}$$

where $\mathbf{u} = (u, v)$ is the in-plane displacement and w is the out-plane displacement, $\mathbf{N}$ and $\boldsymbol{\gamma}$ are the membrane stress and strain. With the vectorial notations $(\mathbf{N} \to {}^t\langle N_x\ N_y\ N_{xy}\rangle, \boldsymbol{\gamma} \to {}^t\langle \gamma_x\ \gamma_y\ 2\gamma_{xy}\rangle)$, the membrane elasticity tensor is represented by the matrix

$$[\mathbf{L}^m] = \frac{Eh}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}. \tag{2}$$

The corresponding internal work $\mathcal{W}_{int}$ can be split into a membrane part $\mathcal{W}_m$ and a bending part $\mathcal{W}_b$ as follows:

$$\begin{cases} \mathcal{W}_{int}(\mathbf{u}, w) = \mathcal{W}_b(w) + \mathcal{W}_m(\mathbf{u}, w), \\ 2\mathcal{W}_b(w) = D\displaystyle\int\int_\omega \left((\Delta w)^2 - 2(1-\nu)\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right)\right)\mathrm{d}\omega, \\ 2\mathcal{W}_m(\mathbf{u}, w) = \displaystyle\int\int_\omega \langle\gamma\rangle[\mathbf{L}^m]\{\gamma\}\mathrm{d}\omega = \frac{Eh}{1-\nu^2}\int\int_\omega (\gamma_x^2 + \gamma_y^2 + 2(1-\nu)\gamma_{xy}^2 + 2\nu\gamma_x\gamma_y)\mathrm{d}\omega. \end{cases} \tag{3}$$

3. Macroscopic membrane model

In this part, a macroscopic finite element model will be presented that is based on a reduced-order modeling introduced in Damil et al. (2013, 2014). The mathematical model has been deduced from the full shell model by the method of Fourier series with slowly variable coefficients (Damil and Potier-Ferry, 2010). The principle of this Fourier-related approach is to write the unknown field in the following form:

$$\boldsymbol{U}(x,y) = \sum_{m=-\infty}^{+\infty} \boldsymbol{U}_m(x,y)e^{miQx}, \tag{4}$$

where the wavenumber Q is given and the macroscopic unknown field $\boldsymbol{U}_m(x,y)$ slowly varies on a period $\left[x, x+\frac{2\pi}{Q}\right]$ of the oscillation. Of course the advantages of such a reduced order model are always counterbalanced by some more or less restrictive assumptions, the main one being here the need of choosing a priori wavenumber. In the case of Föppl–Von Karman equation (1), the unknown field is $\boldsymbol{U}(x,y) = (\mathbf{u}(x,y), w(x,y), \mathbf{N}(x,y), \boldsymbol{\gamma}(x,y))$. Of course, in practice only a finite number of Fourier coefficients will be considered. As pictured in Fig. 1, at least two functions $\boldsymbol{U}_0(x,y)$ and $\boldsymbol{U}_1(x,y)$ are necessary to describe nearly periodic patterns: $\boldsymbol{U}_0(x,y)$ can be identified to the mean value while $\boldsymbol{U}_1(x,y)$ represents the envelope or the amplitude of the spatial oscillations.

In this work, the unknown fields $\boldsymbol{U}(x,y)$ are expressed only in terms of two harmonics: the mean field $\boldsymbol{U}_0(x,y)$ and the first order harmonics $\boldsymbol{U}_1(x,y)e^{iQx}$ and $\overline{\boldsymbol{U}}_1(x,y)e^{-iQx}$. The mean value $\boldsymbol{U}_0(x,y)$ is real-valued, while the other envelops can be complex-valued. So the envelope of the first harmonic $\boldsymbol{U}_1(x,y)$ can be written as $\boldsymbol{U}_1(x,y) = \boldsymbol{r}(x,y)e^{i\varphi(x)}$, where $\boldsymbol{r}(x,y)$ represents the amplitude modulation and $\varphi(x)$ the phase modulation. If the phase linearly varies $(\varphi(x) = qx + \varphi_0)$, this type of approach is able to describe quasi-periodic responses whose wavenumber $Q + q$ slightly differs from the a priori chosen Q. Hence, the method could account for a change of wavenumber.