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Topology optimization of lightweight periodic lattices under simultaneous compressive and shear stiffness constraints

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ABSTRACT

This paper investigates the optimal architecture of planar micro lattice materials for minimum weight under simultaneous axial and shear stiffness constraints. A well-established structural topology optimization approach is used, where the unit cell is composed of a network of beam elements (Timoshenko beams are used instead of truss elements to allow modeling of bending-dominated architectures); starting from a dense unit cell initial mesh, the algorithm progressively eliminates inefficient elements and resizes the essential load-bearing elements, finally converging to an optimal unit cell architecture. This architecture is repeated in both directions to generate the infinite lattice. Hollow circular cross-sections are assumed for all elements, although the shape of the cross-section has minimal effect on most optimal topologies under the linear elasticity assumption made throughout this work. As optimal designs identified by structural topology optimization algorithms are strongly dependent on initial conditions, a careful analysis of the effect of mesh connectivity, unit cell aspect ratio and mesh density is conducted. This study identifies hierarchical lattices that are significantly more efficient than any isotropic lattice (including the widely studied triangular, hexagonal and Kagomé lattices) for a wide range of axial and shear stiffness combinations. As isotropy is not always a design requirement (particularly in the context of sandwich core design, where shear stiffness is generally more important than compressive stiffness), these optimal architectures can outperform any established topology. Extension to 3D lattices is straightforward.

1. Introduction

Metallic cellular materials possess unique combinations of low weight, high stiffness and strength, and enable substantial energy absorption at relatively low crushing stress (Evans et al., 2001, 2010). Additionally, when designed with interconnected porosity, the open volume in the architecture can be exploited for active cooling or energy storage, providing unique opportunities for multifunctionality (Valdevit et al., 2006a; Bell et al., 2005). These attributes make metallic cellular solids uniquely suited as cores of sandwich structures for applications ranging from lightweight aerospace structures to blast-resistant armors (for both land and sea vehicles) (Evans et al., 2010; Wadley et al., 2010), and actively cooled panels for combustor walls of next-generation hypersonic vehicles (Valdevit et al., 2011, 2008). From a mechanical standpoint, the core of a well-designed sandwich panel needs to possess excellent shear stiffness and strength (to support the internal shear force that develops under transverse loads on the panel) as well as compressive stiffness and strength along the through-thickness direction of the panel (to resist indentation under concentrated transverse loads) (Allen, 1969).

At a given relative density (defined as the mass density of the cellular medium divided by the mass density of the solid constituent), topologically architected cellular structures (e.g., periodic architectures) are vastly superior to stochastic foams, by virtue of a more efficient stress transfer mechanism between the macroscale and the unit-cell level: when appropriately designed, each unit-cell element (whether a truss or a shell feature) will largely experience tension or compression under the applied external loads, with minimal bending (Evans et al., 2001; Deshpande et al., 2001). This guarantees full exploitation of the mechanical properties of the base material, providing the cellular material exceptional mechanical efficiency (in terms of specific stiffness and strength). Over the past decade, a number of cellular topologies were investigated and characterized, ranging from truss-like concepts (Deshpande et al., 2001; Zok et al., 2003, 2004) to prismatic (honeycomb-type) designs (Valdevit et al., 2004; Zok et al., 2005). Prismatic designs with the channels in the plane of the sandwich panel (hence

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offering open porosity) can be thought of as two-dimensional topologies, extruded in the third direction. The most common 2D topologies are hexagonal, triangular, and Kagomé designs, regular lattices in which all elements have the same length (Gibson and Ashby, 1999; Christensen, 1995). The effective mechanical properties of these simple lattices are readily extracted in analytical form. Importantly, because of the threefold symmetry, all three designs are in-plane isotropic.

Although isotropy is a desirable property in a number of applications, it is not essential (or even advantageous) for the core of a sandwich panel: appropriately tailoring the anisotropy (e.g., independently choosing compressive and shear stiffness and strength) may in principle result in much more weight efficient designs. Besides isotropy, the choice of periodic architectures with simple unit cells and very few length scales was traditionally justified by manufacturability requirements. Recently, with the development and advancement of a plethora of additive manufacturing techniques (e.g., stereolithography, select laser sintering, direct metal manufacturing (Gibson et al., 2010), SPPW-based manufacturing (Schaedler et al., 2011; Jacobsen et al., 2007)), the ability to fabricate extremely complex and hierarchical architectures has been rapidly growing.

In most studies, optimal designs of lightweight cellular materials have been identified by optimizing the geometric parameters of a predefined lattice-type architecture (Valdevit et al., 2004, 2006b). Although this technique allows analytical description for appropriately chosen topologies, it relies on the intuition of the designer in the selection of the lattice topology. Topology optimization presents a more elegant approach (Cadman et al., 2013). In its classic continuum form, a unit cell is meshed with finite elements, each of which can be assigned either of two phases (e.g., solid and void). The optimizer progressively reassign elements until an optimal phase distribution is achieved. Design of cellular materials has been greatly investigated using topology optimization method, for example, by Sigmund (1995) in design of materials with prescribed mechanical properties, Sigmund and Torquato (1997) in design of multiphase materials for extreme thermal expansion, Silva et al. (1997) in design of piezoelectric microstructures, Dobson and Cox (1999) for design of photonic crystals for band-gaps, and Sigmund and Jensen (2003) for design of materials and structures for phononic band-gaps. Further elaborations of this technique, such as multi-scale optimal design (Liu et al., 2008), analysis of the effects of boundaries (Yan et al., 2006), and optimal design of isotropic cellular solids with prescribed effective moduli and conductivity (Hyun and Torquato, 2002) have been presented. Recently, more complicated materials systems have been analyzed, for example functionally graded materials with desired effective properties (Paulino et al., 2009), and materials with prescribed nonlinear properties (Wang et al., 2014).

Although extremely powerful, continuum topology optimization does not guarantee that the optimal topology be a lattice design. If this is desired, truss-like (or discrete as opposed to continuum) topology optimization is the ideal approach. Starting from a dense mesh of lattice members (Dorn et al., 1964) for a unit cell, truss (or beam)-based topology optimization seeks the best connectivity by removing inefficient elements and resizing the cross-section of the most efficient ones. See Bendsøe and Sigmund (2003) and Rozvany (1996) for more details on topology optimization of truss-like structures. This technique was first applied to the optimization of effective properties of a cellular medium (inverse homogenization) in Sigmund (1994); recently, Asadpoure et al. (2014) extended this approach to integrate the fabrication cost of lattices in the objective function.

In this context, this article numerically investigates the minimum-density designs of periodic 2D lattices under arbitrary combinations of prescribed axial (e.g., compressive) and shear moduli. Optimal lattice architectures are extracted using a formal structural topology optimization algorithm, and the stiffnesses of each design are calculated via the finite element method, utilizing beam elements to model all lattice members. Given the intense recent interest in hollow micro-lattices as an architecture that could provide exceptionally low density and a wide length scale hierarchy (Schaedler et al., 2011; Valdevit et al., 2013; Maloney et al., 2013), in all the calculations the cross-section of each lattice member is assumed to be circular and hollow. However, because most optimal designs support loads primarily by axial deformation (as opposed to bending) of the members, the actual shape of the cross-section has minimal effect on the results (see Section 3.2).

The article is presented as follows. Section 2 defines the minimum relative density problem with axial and shear elastic constraints on a unit cell of the lattice. The unit cell consists of Timoshenko beam elements with hollow circular cross-section, whose existence, thickness, and radius are modeled as continuous design variables, in order to take advantage of gradient-based optimizers. The finite element analysis, including the required boundary conditions for obtaining axial and shear moduli, are presented in Section 2.2. The sensitivity analysis required for the gradient-based optimizer is derived in Section 2.3, followed by the details of the algorithm used for the topology optimization in Section 2.4. Optimized solutions, compared to the well-known bounds on isotropic cellular materials and with the most commonly available 2D lattices (triangular, hexagonal and Kagomé designs), are presented in Section 3. In the same section, the effects of lattice hierarchy is discussed. Conclusions follow. The appendices include a mesh sensitivity analysis, discussing the effect of initial mesh density, domain shape and upper bound on the lattice member radius.

2. The topology optimization problem

2.1. Problem statement

The objective of the optimization is to find the minimum weight of a two-dimensional periodic lattice material under simultaneous axial and shear stiffness constraints, i.e. the optimized lattice maintains a minimum axial stiffness as well as a minimum shear stiffness. A structural topology optimization algorithm is used. The unit cell of the lattice is initially seeded with a dense mesh of structural finite elements; beam elements are used as opposed to truss elements, in order to allow load carrying by bending rather than solely by axial deformation. Although optimally designed lattices are almost always statically determinate (and hence carry load by axial deformation of each member), allowing for bending deformation might be important for extremely anisotropic designs where the required axial and shear stiffness are vastly different. As the optimization procedure progresses, inefficient elements are eliminated and the cross-sections of the remaining elements are resized, ultimately converging to the optimal minimum-density lattice architecture. A binary design variable, x_x^e, is assigned to each lattice element to represent its existence (i.e., $x_x^e = 1$ if the element e exists, otherwise $x_x^e = 0$). The need for the introduction of this additional variable is explained later in this section. The formal optimization problem on a discretized domain Ω (representing a unit cell or fraction thereof) can be expressed as follows:

$$\min_{\mathbf{x}} \ \overline{\rho}(\mathbf{x}) = \sum_{\forall e \in \Omega} \frac{x_x^e v^e(\mathbf{x}_c)}{V^\Omega} \tag{1}$$

$$s.t. \ C_E^\Omega(\mathbf{x}) \leqslant C_E^* \tag{2}$$

$$C_G^\Omega(\mathbf{x}) \leqslant C_G^* \tag{3}$$

$$x_x^e = \begin{cases} 1 & \text{if solid} \\ 0 & \text{if void} \end{cases}, \quad \forall e \in \Omega \tag{4}$$