International Journal of Solids and Structures 60–61 (2015) 28–34

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Behaviour of porous ductile solids at low stress triaxiality in different modes of deformation

Viggo Tvergaard

Department of Mechanical Engineering, Solid Mechanics, Technical University of Denmark, DK-2800 Kgs., Lyngby, Denmark

ARTICLE INFO

Article history:
Received 18 November 2014
Received in revised form 12 January 2015
Available online 7 February 2015

Keywords:
Voids
Low stress triaxiality
Large strain plasticity
Contact
Pure shear

ABSTRACT

The effect of low stress triaxiality on ductile failure is investigated for a material subject to pure shear or to stress states in the vicinity of pure shear. Many recent studies of ductile failure under low hydrostatic tension have focused on shear with superposed tension, which can result in simple shear or in stress states near that. A material with a periodic array of voids is subjected to tensile stresses in one direction and compressive stresses in the transverse direction. Numerical solutions for a plane strain unit cell model are obtained numerically. For stress states in the vicinity of pure shear it is found that the voids close up to micro-cracks, and these cracks remain closed during continued deformation, with large compressive stresses acting between crack surfaces. The same type of behaviour is found for different initial sizes of the voids and for cases where the two types of voids in the unit cell have very different initial size. The analyses do not indicate a final failure mode where the stress carrying capacity of the material drops off to zero. In previous analyses for stress states in the vicinity of simple shear such final failure has been predicted, so it appears that the behaviour of a porous ductile material at low stress triaxiality depends a great deal on the mode of deformation.

1. Introduction

Under relatively high hydrostatic tension micro-voids contained in a ductile material will tend to grow large during plastic deformation, and ductile failure will occur by coalescence of neighbouring voids (see reviews by Garrison and Moody, 1987; Tvergaard, 1990; Benzerga and Leblond, 2010). However, recently there has been increasing interest in the behaviour of voids under low stress triaxiality. Barsoum and Faleskog (2007a) have carried out full 3D analyses for shear specimens containing spherical voids in order to model their experiments (Barsoum and Faleskog, 2007b) on ductile fracture in a double notched tube specimen loaded in combined tension and torsion. In a number of plane strain cell model analyses for a material containing a periodic array of circular cylindrical voids Tvergaard (2008, 2009, 2012) and Dahl et al. (2012) have shown that in stress states similar to simple shear the voids are flattened out to micro-cracks, which rotate and elongate until interaction with neighbouring micro-cracks gives coalescence, and this mechanism has also been found in 3D for initially spherical voids (Nielsen et al., 2012). Thus, under high stress triaxiality the void volume fraction increases until ductile fracture occurs, whereas the void volume fraction disappears under low stress triaxiality, as the voids become micro-cracks. In analyses of cases where micro-cracks form it is important to account for the contact between crack surfaces.

An extension of the Gurson model has been proposed by Nahshon and Hutchinson (2008) to be able to describe failure in simple shear where the hydrostatic tension is zero. In this extended model the damage parameter is no longer a geometrically well defined void volume fraction, so this aspect of the model is more like continuum damage mechanics. Tvergaard and Nielsen (2010) have compared predictions of this shear-extended Gurson model with predictions of the micro-mechanical studies (Tvergaard, 2009) and have found that the trends of the predictions are in good agreement.

A number of recent experimental investigations have considered ductile fracture in shear at a stress triaxiality near zero. Thus, Bao and Wierzbicki (2004), Beese et al. (2010) and Dunand and Mohr (2011), for two different aluminium alloys and a TRIP steel, have used special butterfly specimens to study the effect of the stress triaxiality and of the Lode angle in stress states dominated by shear. Haltom et al. (2013) have used a tubular specimen in tension–torsion while Ghahremaninezhad and Ravi-Chandar (2013) have used a modified Arcan test to study the same Al 6061-T3.

In the investigations mentioned above it is characteristic that plastic deformations under low hydrostatic tension are studied

E-mail address: viggo@mek.dtu.dk

http://dx.doi.org/10.1016/j.ijsolstr.2015.01.027
0020-7683/

under shear loading. When the hydrostatic tension is precisely zero, this mode of deformation is called simple shear. However, deformation under low stress triaxiality can also be applied by subjecting the material to tensile loading in a fixed direction, while compressive loading is applied in the transverse direction. Then material lines along these two loading directions do not at all rotate during the plastic deformations. When the hydrostatic tension is precisely zero, this mode of deformation is called pure shear.

In the present paper a material containing a periodic array of voids is studied under tension in one fixed direction and compression in the transverse direction, so that the modes of deformation considered are either pure shear or in the vicinity of pure shear. The material response is analysed by numerical solutions for a characteristic unit cell model. The main purpose here is to investigate whether or not ductile failure is predicted in these stress states similar to pure shear, analogous to the predictions for stress states similar to simple shear, where the voids are flattened out to micro-cracks, which elongate until interaction with neighbouring micro-cracks gives coalescence.

2. Problem formulation and numerical procedure

The material to be considered here (Fig. 1) has a periodic array of voids, with the initial spacing A_0 in the x^1-direction and the initial spacing B_0 in the x^2-direction. Plane strain conditions are assumed. The voids are initially circular cylindrical and staggered, with the radii R_{01} and R_{02}, respectively, so that the voids are located in opposite corners of the rectangular unit cell in Fig. 1. Finite strains are accounted for and the analyses are based on a convected coordinate Lagrangian formulation of the field equations, with a Cartesian x^i coordinate system used as reference and with the displacement components on reference base vectors denoted by u^i. The metric tensors in the reference configuration and the current configuration, respectively, are g_{ij} and G_{ij} with determinants g and G, and $\eta_{ij} = 1/2(G_{ij} - g_{ij})$ is the Lagrangian strain tensor. In terms of the displacement components u^i on the reference base vectors the Lagrangian strain tensor is

$$\eta_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u^k_{,i}u_{k,j}\right) \tag{1}$$

where $(\ )_{,j}$ denotes covariant differentiation in the reference frame. The contravariant components τ^{ij} of the Kirchhoff stress tensor on the current base vectors are related to the components of the Cauchy stress tensor σ^{ij} by $\tau^{ij} = \sqrt{G/g}\,\sigma^{ij}$. A finite strain formulation for a J_2 flow theory material with the Mises yield surface is applied, where the incremental stress–strain relationship takes the form $\dot{\tau}^{ij} = L^{ijk\ell}\dot{\eta}_{k\ell}$, with the instantaneous moduli specified in

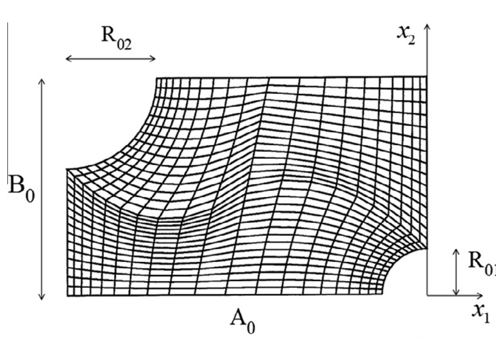

Fig. 1. A unit cell used to analyse a material with a periodic array of circular cylindrical voids. Example of an initial mesh is shown for a case with $B_0/A_0 = 0.6$, $R_{01}/A_0 = 0.125$ and $R_{02}/A_0 = 0.25$.

(Hutchinson, 1973; Tvergaard, 1976). The true stress-logarithmic strain curve in uniaxial tension is taken to follow the power law

$$\varepsilon = \begin{cases} \sigma/E, & \sigma \leqslant \sigma_Y \\ (\sigma_Y/E)(\sigma/\sigma_Y)^{1/N}, & \sigma \geqslant \sigma_Y \end{cases} \tag{2}$$

with Young's modulus E, the initial yield stress σ_Y and the power hardening exponent N. Poisson's ratio is ν.

The material is subjected to tensile loading in the x^2-direction, such that average true stress Σ_{22} in the vertical direction is a principal stress. Thus, also the average true stress Σ_{11} in the x^1-direction is a principal stress. The numerical calculations are carried out such that a fixed stress ratio is prescribed

$$\Sigma_{11}/\Sigma_{22} = \kappa \tag{3}$$

and in most cases the value of the constant κ is prescribed as negative (where the value -1 corresponds to pure shear). With the assumed symmetries of the material geometry and with the loading applied in the vertical and horizontal directions the material lines along the coordinate axes will remain straight throughout the deformation. Thus, the boundary conditions to be satisfied along all four edges of the rectangular unit cell are standard symmetry conditions. The average logarithmic strains in the two coordinate directions are denoted ε_1 and ε_2, respectively.

When the hydrostatic tension is sufficiently low in the present computations, the voids are going to close up to form micro-cracks, so that contact conditions are needed for the points on the void surface. For each nodal point on the void surface the displacements are checked relative to the location of the edge lines crossing the void. If the displacements exceed such a symmetry line, this surface point overlaps with the symmetrically located surface point in the neighbouring unit cell, and this marks the onset of surface contact at that particular nodal point. From then on the displacement normal to the edge of the unit cell is prescribed to be equal to that of the edge line. Subsequently, when contact has been established in a nodal point on the void surface, the value of the compressive nodal force on the symmetry plane is checked, and if this force becomes tensile, the contact is released, so that the void can start opening up again. Friction during contact is not an issue here, as the symmetry line through each void means, that there is no sliding between the void surfaces.

It is noted that initially computations have been carried for only half the unit cell, i.e. a cell with the length $A_0/2$ in the x^1-direction and the height B_0 in the x^2-direction, containing only part of one void. However, the staggered void arrangement in Fig. 1 has been preferred, since this promotes interaction between the voids by a region of intense shear strains, and this also allows for considering different size voids.

The numerical solutions for the fields inside the unit cell are obtained by a linear incremental solution procedure, based on the incremental principle of virtual work. On the void surfaces zero nominal tractions are specified, until contact occurs as described above. The displacement fields are approximated in terms of 8-noded isoparametric elements, and volume integrals in the principle of virtual work are carried out by using 2×2 point Gauss integration within each element. An example of a mesh used for some of the numerical analyses is shown in Fig. 1.

In each increment an increasing average strain ε_2 is prescribed in the vertical direction, and the increment of the transverse strain is calculated such that the prescribed stress ratio (3) remains satisfied. This is carried out by using a Rayleigh Ritz-finite element method (Tvergaard, 1976).

Remeshing is used a few times in each computation to avoid severe mesh distortion. The remeshing procedure applied was first introduced in one of the authors finite strain programmes by Pedersen (1998), and has been further developed in (Tvergaard,