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A refined micromechanical damage–friction model with strength prediction for rock-like materials under compression

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ABSTRACT

Inelastic deformation and damage evolution at microdefects are two essential nonlinear mechanisms that govern macroscopic mechanical behaviors of quasi-brittle solids. The present paper deals in a unified framework with two dissipative processes in microcracks: inelastic deformation due to frictional sliding and damage by crack growth, usually arising and strongly coupled in cohesive materials under compression. Contributions by this work are threefold: (i) based on the Mori–Tanaka method, the free enthalpy of the representative elementary volume composed of a matrix phase and randomly oriented and distributed penny-shaped microcracks is determined for the general case of multiple crack families. The constitutive formulations are now presented in an elegant manner by using two orientation-dependent tensorial operators; (ii) the friction criterion is formulated in terms of the local stress applied onto microcracks. This local stress contains a back stress term that allows unified modeling of material hardening/softening behavior: friction-induced hardening is attributed to the cumulation of frictional shearing while damage-related softening is induced by crack growth and coalescence; (iii) originally, strength prediction is achieved through damage–friction coupling analyses. In that process, a basic feature of the damage resistance is revealed, leading to a novel damage criterion suitable for describing and modeling nonlinear mechanical behavior of quasi-brittle materials. Moreover, trans-scale relationship between the parameters in the local criteria and experimental data from laboratory tests is set up, which is always appealing in multiscale modeling. As a first phase of validation, the refined micromechanical model is finally applied to simulate laboratory tests on a granite under triaxial compression.

1. Introduction

When subjected to compressive loads, cohesive-frictional solids (e.g., concrete, rocks and ceramics) present nonlinear mechanical behaviors and induced material anisotropy. In that context, two main dissipative mechanisms have been widely identified: damage by crack growth and frictional sliding along closed crack surfaces, which are usually strongly coupled to each other. Theoretical investigations have shown that the damage–friction coupling analysis allows explaining and modeling quite satisfactorily main experimental phenomena observed in quasi brittle materials, such as the nonlinearity of mechanical response, induced anisotropy, effect of confining pressure, volumetric dilatancy, hysteresis during unloading–reloading, material hardening/softening.

In the context of micromechanics-featured damage modeling, some theoretical models based on direct approaches have first been developed (Kachanov, 1982; Andrieux et al., 1986; Gambarotta and Lagomarsino, 1993; Prat and Bazant, 1997; Pensée et al., 2002; Jefferson and Bennett, 2007). It is remarked that these models rely on the basic results established in the Linear Elastic Fracture Mechanics without using rigorous upscaling methods. Their extension to take into account more general conditions (e.g., complex loadings, multiple interacting cracks, multiphysical coupling) has met more or less theoretical difficulties. Alternatively, use of the standard Eshelby-based homogenization procedure for heterogeneous materials has been made to describe the mechanical and poromechanical behaviors of cracked solids (Barthelemy et al., 2003; Dormieux et al., 2006). Along this line, Zhu et al. (2008a) incorporated the linear homogenization estimates into the standard framework of irreversible thermodynamics. This choice has facilitated the constitutive formulation and also numerical implementation into finite element codes for structural analyses

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Notation

$\boldsymbol{n}$	unit normal vector for a family of microcracks
$\mathbb{N}$	$= \boldsymbol{n}\otimes\boldsymbol{n}\otimes\boldsymbol{n}\otimes\boldsymbol{n}$ fourth order normal operator
$\mathbb{T}$	fourth order tangential operator
$\boldsymbol{\delta}$	second order unit tensor
$\mathbb{I}$	fourth order unit tensor
$\mathbb{C}^m$	isotropic elasticity tensor of the matrix phase
$\mathbb{S}^m$	isotropic compliance tensor of the matrix phase
$\mathbb{C}^{hom}$	homogenized (effective) elasticity tensor of the RVE
$\mathbb{S}^{hom}$	homogenized (effective) compliance tensor of the RVE
E^m	Young's modulus of the matrix phase
ν^m	Poisson's ratio of the matrix phase
$\boldsymbol{\sigma}$	macroscopic stress tensor
$\boldsymbol{\varepsilon}$	macroscopic strain tensor
$\boldsymbol{\varepsilon}^m$	mean elastic strain in the matrix phase
$\boldsymbol{\varepsilon}^c$	total inelastic strain induced by microcracks
$\phi^{c,r}$	volume fraction of microcracks in the rth family
d^r	crack density parameter serving as internal damage variable
$\boldsymbol{\sigma}^{c,r}$	local stresses applied onto microcracks in the rth family
$\boldsymbol{\epsilon}^{c,r}$	local inelastic strain induced by microcracks in the rth family
$\mathbb{S}^{n,r}$	modification by microcracks to $\mathbb{S}^m$ for the unit damage
σ_n^c	normal projection of the local stress vector ($\boldsymbol{\sigma}^c.\boldsymbol{n}$)
$\boldsymbol{\tau}^c$	tangential projection of the local stress vector ($\boldsymbol{\sigma}^c.\boldsymbol{n}$)
η	coefficient of friction of crack surfaces
$\mathcal{R}(d)$	current material resistance to damage evolution
θ_c	inclined angle of the critical failure plane
d_c	critical damage corresponding to the material failure
r_c	maximum damage resistance
ϕ	intrinsic friction angle of the material

(Zhu et al., 2008b), and thus opens a new and promising way to investigate multi-mechanism coupling behaviors of microcracked media as well as engineering application of the constitutive equations. From then on, some important extensions have been made to several aspects. We first mention the isotropic unilateral damage–friction model (Zhu et al., 2011) and its application to brittle rocks (Xie et al., 2011) as well as its poromechanical extension by Xie et al. (2012) for saturated porous materials. On the other hand, multiphysical coupling models have been developed in the same framework, for instance, damage-permeability coupling by Jiang et al. (2010) and Chen et al. (2014), damage-induced thermomechanical coupling by Chen et al. (2012).

Although significant progress has been made in multiscale modeling of cracked solids, there still need in-depth studies on damage–friction coupling and relevant theoretical issues. In fact, some important issues remain largely open by now, for example, the determination of damage criterion under a purely micromechanical background, and strength prediction in the two competing and coupled dissipative processes particularly in the material containing multiple families of microcracks which generally evolve non-uniformly under complex loading conditions.

This paper aims at presenting a refined micromechanical anisotropic unilateral damage model for quasi-brittle geomaterials. In the previous work (Zhu et al., 2008a), although the effective free energy and then free enthalpy by applying the Legendre–Fenchel transformation have been determined (Zhu et al., 2008b), the extension to the general case of multiple crack families was not rational but simply operated by summation. Moreover, the previous model failed in predicting material strength and modeling damage softening. The objective of this paper is threefold: firstly, based on the Mori–Tanaka homogenization scheme, we determine in a more consistent way the free enthalpy of the representative elementary volume that is composed of a matrix phase and randomly oriented and distributed penny-shaped microcracks. The constitutive equations are reformulated in an elegant manner by introducing two orientation-dependent tensorial operators; secondly, perform strength prediction based on damage–friction coupling analyses. In that process, basic relationship between the parameters involved in the local criteria and experimental data from laboratory tests is originally set up in order to facilitate the calibration of the model's parameters. The last purpose is to reveal the basic feature of the damage resistance function and to develop a novel damage criterion suited for describing and modeling damage-induced softening behaviors of quasi-brittle materials. In order to assess its predictive ability, the refined model is finally applied to simulate triaxial compression tests on a granite.

Throughout the paper, the following notation on tensor products of any two second-order tensors $\boldsymbol{A}$ and $\boldsymbol{B}$ will be used: $(\boldsymbol{A}\otimes\boldsymbol{B})_{ijkl} = A_{ij}B_{kl}$ and $(\boldsymbol{A}\underline{\overline{\otimes}}\boldsymbol{B})_{ijkl} = (A_{ik}B_{jl} + A_{il}B_{jk})/2$. The tensor product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is denoted by $(\boldsymbol{a}\otimes\boldsymbol{b})_{ij} = a_i b_j$ and its symmetric part is taken as $(\boldsymbol{a}\otimes^s\boldsymbol{b})_{ij} = (a_i b_j + a_j b_i)/2$.

2. Description of a matrix-cracks system

Upscaling analyses are usually performed over a representative elementary volume (REV) that is assumed to occupy a geometric domain Ω limited by its external boundary surface $\partial\Omega$. We are concerned here with the REV composed of an isotropic linearly elastic matrix with elasticity tensor $\mathbb{C}^m$ and of a large number of penny-shaped microcracks. In that context, the presence of microcracks weakens the solid matrix and crack growth in preferred directions usually induce material anisotropy. To carry out micromechanical analyses based on the Eshelby's solution to the well-known equivalent inclusion problem, the REV is viewed as a matrix-inclusion system. More precisely, microcracks are modeled as ellipsoidal inclusions embedded in the solid matrix and assumed to potentially propagate in a self-similar manner. Further, all microcracks with the same normal vector are put into the same family. Thus, a family of oblate ellipsoids (penny-shaped cracks) can be sufficiently characterized by its normal $\boldsymbol{n}$, mean radius a and mean half opening c (see Fig. 1). The volume fraction of this family of cracks, denoted ϕ^c is then given by $\phi^c = \frac{4}{3}\pi a^2 c\mathcal{N} = \frac{4}{3}\pi\varsigma d$ where $\varsigma = c/a$ represents the aspect ratio of cracks, $\mathcal{N}$ denotes the number of cracks per unit volume, and $d = \mathcal{N}a^3$ is the well-known crack density parameter and most often directly used as an internal damage variable.

2.1. Inelastic deformation by microcracks

Consider a macroscopic uniform strain $\boldsymbol{\varepsilon}$ applied to the boundary $\partial\Omega$. The main purpose of multiscale analyses is to derive the fields of stress $\boldsymbol{\sigma}$, strain $\boldsymbol{\varepsilon}$ and displacement $\boldsymbol{u}$ inside the REV as well

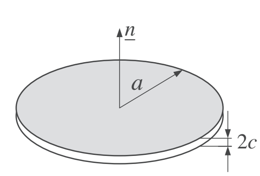

Fig. 1. Geometric description of a penny-shaped microcrack.