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Isotropic to distortional hardening transition in metal plasticity

Niko Manopulo [a,*], Frédéric Barlat [b], Pavel Hora [a]

[a] Institute of Virtual Manufacturing, Swiss Federal Institute of Technology (ETH Zurich), Tannenstrasse 3, 8092 Zurich, Switzerland
[b] On leave from Graduate Institute of Ferrous Technology (GIFT), Pohang University of Science and Technology (POSTECH), San 31, Hyoja-dong, Nam-gu, Pohang, Gyeongbuk 790-784, Republic of Korea

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ABSTRACT

The present paper aims to discuss the transition from isotropic to distortional hardening behavior of metallic materials, based on the Homogeneous Anisotropic Hardening (HAH) model. Furthermore, the effect of yield locus distortion on the evolution of the strain increment, under the assumption of associated flow, is theoretically discussed and exemplified. Special cases, such as coaxial and orthogonal stress states, are analyzed to provide better insight into the model. Particular emphasis is put on the monotonic loading case, which is compared to isotropic hardening. Finally, the evolution equations of the state variables are examined and their properties are discussed.

1. Introduction

The materials used in automotive industry are rapidly evolving. Policy decisions for manufacturing energy efficient products on the one hand, and continuously increasing requirements on passenger safety on the other, drive the industry towards using materials with a high strength-to-weight ratio. This, in turn, spurns the development of Advanced High Strength Steels (AHSS) as well as lighter metals such as aluminum and magnesium alloys. The latter not only exhibit strong initial anisotropy, they are also subject to significant loading path dependence. Microstructure driven mechanisms such as the Bauschinger effect and latent hardening/softening, thus become important and need to be considered in modeling these materials.

Distortional hardening models have received attention since many years, early approaches such as the ones proposed Ortiz and Popov (1983) and Voyiadjis and Foroozesh (1990) described a change in the shape of the yield locus with plastic deformation. A more complex model has been proposed by Kurtyka and Zyczkowski who considered affine deformation and rotation in addition to proportional expansion, translation and distortion (Kurtyka and Zyczkowski, 1996). Another comprehensive model, including isotropic, kinematic and directional hardening effects has been proposed in a series of works by Feigenbaum and Dafalias (2007, 2008) and Dafalias and Feigenbaum (2011) as well as Aretz (2008). More recently Shutov and Ihlemann suggested a viscoplasticity approach for the description of distortional hardening phenomena (Shutov and Ihlemann, 2012). Experimental analysis of distortional hardening has been published by Phillips et al. (1972) and more recently in two contributions by Khan et al., for an aluminium alloy deformed under proportional and non proportional loading conditions (Khan et al., 2009, 2010). Subsequently, Pietryga et al. used a finite deformation model to investigate these effects and discussed their agreement with experiments (Pietryga et al., 2012).

There have been numerous efforts in modeling the Bauschinger effect. A comprehensive review can be found in Chaboche (2008). Modelling approaches for the specific case of sheet forming can be found in Yoshida et al. (2002), Yoshida and Uemori (2002) and Yoshida and Uemori (2003). A tensorial description of dislocation structures which evolve with plastic deformation has been proposed by Teodosiu and Hu (1998). This effect has been also thoroughly investigated at the crystal plasticity level (see e.g. by Peeters et al. (2000, 2001), Franz et al. (2009) and Kitayama et al. (2013)).

The Homogeneous Anisotropic Hardening (HAH) model has been first proposed in 2011 (Barlat et al., 2011) for capturing the Bauschinger effect and subsequently extended and revised to include latent effects (Barlat et al., 2013, 2014). In contrast to earlier methods, which are mostly based on the concept of kinematic hardening, the HAH model provides a modular framework for the description of anisotropy, Bauschinger and latent effects, either independently or combined in an arbitrary manner. This releases restrictions about the use of particular yield loci or hardening models on the one hand and enables the extension of

* Corresponding author.
E-mail address: manopulo@ivp.mavt.ethz.ch (N. Manopulo).

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existing models in order to account for microstructural effects on the other.

The HAH model has been used in the recent literature in order to investigate the role of microstructure effects on different processes. Lee et al. (2012) used the model to investigate the role of the Bauschinger effect on springback for U-draw/bending of pre-strained material. Lee et al. (2012) compared the accuracy of springback prediction of the HAH model with that of the Chaboche kinematic hardening model. He et al. (2013) proposed an extension to the original HAH model able to capture work hardening stagnation and cross-effects, by introducing four new parameters. A meso-scale simulation approach has been proposed by Ha et al. (2014) which uses the model in combination to a crystal plasticity approach in order to investigate the behavior of dual-phase steels. The effect of continuous versus abrupt strain path change has been investigated in another contribution by Ha et al. (2014), who also tested the ability of the model to capture the rate of path change from plane strain to simple shear. The effect of cyclic loading on fatigue prediction of low carbon sheet steel has been studied by Hariharan et al. (2014). Lee et al. introduced a dislocation density based hardening model into the HAH framework and tested the results related to springback accuracy. Similarly Lee et al. (2013) proposed an extension of the quasi-plastic elastic approach by incorporating nonlinear elasticity effects to improve the accuracy in springback prediction.

The present contribution primarily aims to discuss, based on the HAH model, the transition between isotropic and distortional hardening responses in case of non-proportional loading. In addition, it is rigorously proven, that the response of the HAH model for the case of proportional loading is strictly identical to that of the classical anisotropic yield function under isotropic hardening. This is valid for the general case, including asymmetric yield loci, with the only restriction being the homogeneity of the stable component, as this is a defining property of the HAH model itself. The proof is achieved through a semi-geometric re-interpretation of the model, which in turn provides insight into the different model features and is expected to facilitate the understanding and application of the methodology.

2. Interpretation of original HAH model

In this section, the different state variables and coefficients controlling the Bauschinger effect in the HAH approach will be revised and investigated, in order to provide better insight into the model. The evolution laws for these variables are reviewed in Section 5.

2.1. Yield condition

The HAH yield condition can be written as follows:

$$\bar{\sigma}^q(\mathbf{s}) = \phi^q(\mathbf{s}) + f_-^q \phi_-^q(\mathbf{s}) + f_+^q \phi_+^q(\mathbf{s}) = \sigma_r^q \tag{1}$$

where ϕ is any positively homogeneous yield function, $\mathbf{s}$ is the stress deviator, q is a blending exponent and σ_r is a reference stress (e.g. uniaxial tensile stress). The state variables f_+ and f_- [1] enable the distortion of the yield locus and must stay positive in order to guarantee convexity of the yield surface. The functions ϕ_+ and ϕ_- quantify the strength of load reversal and are defined as follows:

$$\phi_- = \left|\hat{\mathbf{h}} : \mathbf{s} - \left|\hat{\mathbf{h}} : \mathbf{s}\right|\right|$$
$$\phi_+ = \left|\hat{\mathbf{h}} : \mathbf{s} + \left|\hat{\mathbf{h}} : \mathbf{s}\right|\right| \tag{2}$$

[1] Note that the notation in Eq. (1) slightly differs from previous publications. The correspondence is such that $f_- = f_1$ and $f_+ = f_2$ and similarly for the other state variables.

where the second order tensor $\hat{\mathbf{h}}$ is called the microstructure deviator and captures the loading history of the material. Its initial value corresponds to the normalized stress deviator at the first plastic strain increment and subsequently evolves according to the laws given in Section (5). Furthermore, this quantity is defined in a manner that its size (as measured by the norm $x = \|\mathbf{x}\| = \sqrt{\mathbf{x} : \mathbf{x}}$) always corresponds to $1/\sqrt{H}$. The normalization parameter H simply represents an arbitrary size and does not influence the model. A normalized version of the stress deviator is also similarly defined:

$$\hat{\mathbf{s}} = \frac{\mathbf{s}}{\sqrt{H\mathbf{s} : \mathbf{s}}} = \frac{1}{\sqrt{H}} \frac{\mathbf{s}}{\|\mathbf{s}\|} = \frac{1}{s\sqrt{H}}\mathbf{s} \tag{3}$$

2.2. Interpretation of $\hat{\mathbf{h}} : \mathbf{s}$

The central quantity in the formulation is the double dot product between the microstructure deviator $\hat{\mathbf{h}}$ and the deviatoric stress tensor $\mathbf{s}$. Geometrically speaking, this can be interpreted as the projection of the stress deviator on the axis defined by the microstructure deviator. This, in turn, provides twofold information about the current stress state with respect to the loading history:

Sign of $\hat{\mathbf{h}} : \mathbf{s}$.

- If $\hat{\mathbf{h}} : \mathbf{s} > 0$ the loading path may have changed direction, but loading has not been reversed. New slip systems are activated but none of the already active slip systems are likely working in the reverse direction.
- If $\hat{\mathbf{h}} : \mathbf{s} < 0$ New slip systems are activated but also some of the active slip systems are active in the reverse direction.
- If $\hat{\mathbf{h}} : \mathbf{s} = 0$ cross loading. None of the previously active slip systems are active in the second deformation step.

Size of $\hat{\mathbf{h}} : \mathbf{s}$.

The quantity $\hat{\mathbf{h}} : \mathbf{s}$ is also related to the "angle" χ between the two tensors. In fact, by using the definition of the angle between two tensors, the latter is defined as:

$$\cos\chi = \frac{\hat{\mathbf{h}} : \mathbf{s}}{\|\hat{\mathbf{h}}\|\|\mathbf{s}\|} = \frac{\sqrt{H}}{s}\hat{\mathbf{h}} : \mathbf{s} = H\hat{\mathbf{h}} : \hat{\mathbf{s}} \tag{4}$$

It is worthwhile to note that the stress deviator can change direction arbitrarily and even abruptly, whereas the microstructure deviator only evolves progressively, as a function of the plastic work, and realigns itself along the stress deviator axis. This property of the model ensures that the yield locus shape remains primarily a function of the prior deformation history and not of the instantaneous stress state. To clarify this point let us assume that $\hat{\mathbf{h}}$ and $\mathbf{s}$ are diagonal and that $\hat{\mathbf{h}} : \mathbf{s} = 0$ at the instant of the load change (i.e. cross-loading). If there is latent hardening, this change translates physically by an overshooting of the flow stress with respect to the monotonic hardening curve. If the new load is such that, instead of being diagonal, $\mathbf{s}$ contains shear components, $\hat{\mathbf{h}} : \mathbf{s}$ is still equal to 0. This means that, at the very moment of the change, the yield surface is the same for these two cases. However, the actual material response will depend on the actual stress applied in the second segment, i.e., with or without shear components. In addition, the absolute value of the off-diagonal components of h starts to increase but only gradually because the microstructure does not change suddenly.

2.3. Interpretation of ϕ_+ and ϕ_-

The HAH model in (1) is devised in a manner that depending on the sign of $\hat{\mathbf{h}} : \mathbf{s}$, only one of the functions ϕ_+ and ϕ_- is active. In fact it is easily seen in Eq. (2), that the following holds: