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Wave propagation in multistable magneto-elastic lattices

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ABSTRACT

One dimensional (1D) and two dimensional (2D) magneto-elastic lattices are investigated as examples of multistable, periodic structures with adaptive wave propagation properties. Lumped-parameter lattices with embedded permanent magnets are modeled as point magnetic dipole moments, while elastic interactions are described as axial and torsional springs. The equilibrium configurations for the lattices are identified through minimization of the lattice potential energy. Bloch wave analysis is then conducted for small perturbations about stable equilibria to predict corresponding wave propagation properties. Finally, nonlinear dynamic simulations validate the findings of the linearized unit cell analysis, and illustrate the changes in dynamic behavior caused by topological transitions. Case studies for 1D systems show how pass bands and bandgaps are defined by lattice reconfigurations and by changes in lattice magnetization. In 2D systems, hexagonal lattices transition from regular honeycombs to re-entrant ones, which leads to significant changes in wave speeds, and directionality of wave motion and transition fronts.

1. Introduction

Periodic cellular structures and lattices are at the object of intense research because of their often advantageous stiffness-to-weight ratios (Schaedler et al., 2013). These structural assemblies not only make an efficient use of material, but are also characterized by interesting dynamic and wave propagation properties. For example, periodic lattices exhibit dispersive behavior, causing different frequency waves to travel at different speeds through the lattice, and are characterized by the occurrence of related "wave beaming" phenomena (Ruzzene et al., 2003). Bandgaps can also occur that prevent waves at certain frequencies to propagate through the system. The aforesaid wave properties are highly dependent on lattice topology as well as stiffness and mass distributions. In the context of this paper, the word "topology" refers to the geometric configuration of a periodic lattice. Changing the topology and stiffness/mass distribution within a lattice therefore provides the means of tailoring and controlling wave motion and the subsequent onset of vibrations. Wave propagation control finds application in methods for isolation of vibrations in rotating machinery (Olson et al., 2014), for the protection of important locations (such as the brain) from waves resulting from blasts and impacts through their steering and redirection (Barsoum, 2011), for the harvesting of mechanical energy and its conversion for power generation by forming wave lenses and reflectors (Carrara et al., 2012), or for the processing of acoustic wave signals (Vasseur et al., 2011).

The topological dependence of wave motion in a lattice has been extensively investigated. For example, differences of in-plane wave propagation between hexagonal and re-entrant lattices were investigated in Gonella and Ruzzene (2008). Similarly, the volume fraction and skewness of rhombic grid lattices were shown to greatly affect wave speed and directionality in Casadei and Rimoli (2013). As structures transition from near-continuum to beam lattices, wave beaming is a prominent phenomena that arises. Furthermore, increasing the skewness of the lattice affects the direction in which beaming occurs. Rotation of lattice components can be exploited to tune bandgaps in a reversible way (Bertoldi and Boyce, 2008; Goffaux and Vigneron, 2001; Li et al., 2003). Specifically, in Bertoldi and Boyce (2008) the topology of a highly deformable 2D structure changes as it buckles under a compressive load. Periodic 2D systems of high-density rectangular inclusions in a fluid are investigated in Goffaux and Vigneron (2001) and Li et al. (2003), where the rotation of the high-density inclusions is the mechanism used to tune bandgaps.

All of the aforementioned studies demonstrate that large changes in wave propagation can be induced by changing lattice topology. The concept of adapting the mechanical wave properties

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of a structure through topology reconfiguration is of interest (Bertoldi and Boyce, 2008; Goffaux and Vigneron, 2001; Li et al., 2003), as it leads to the design of components with adaptive dynamic behavior. In this context, self-stable topologies featuring multiple equilibrium configurations are of particular interest. Structures exhibiting more than one self-stable topology allow switching between operational modes associated with different dispersion, bandgap, and beaming properties (i.e. wave propagation properties), without the need for the continued application of an external control action. Such topological reconfiguration may allow, for example, a filter that can switch between two operational frequencies, or one structure that can turn a function on and off. Previous studies of relevance include the work of Vasseur et al. (2011), which provides a good example of how changes in stiffness (or elastic moduli) of magnetostrictive materials can be used to affect wave motion. Another tunable magneto-elastic system consists of magnetostrictive particles immersed in an elastic medium, as investigated in Yin et al. (2006). Analogous topology and stiffness changes are applied in reconfigurable electromagnetic wave filters, as illustrated for example in Park et al. (2005) and Karim et al. (2006). These devices reconfigure their circuit geometry through the activation of electrical switches, allowing a change in the frequencies that can propagate. The periodic structure presented in Karim et al. (2006) uses PIN diodes as switches to turn the propagation of an electromagnetic wave on or off. Also, the devices in Park et al. (2005) reconfigure through the actuation of micro-mechanical switches, adjusting the geometry of the electromagnetic device to toggle the operational frequency of low-pass and band-pass filters.

Magneto-elastic systems generally have the ability to be multistable and self-stable (Moon and Holmes, 1979; Harne and Wang, 2013). This makes them good candidates for the design of periodic lattices with topologies that can be toggled (Lapine et al., 2011). Drastic topological changes in a magneto-elastic lattice are demonstrated experimentally in Tipton et al. (2012), where topological transformations similar to those in Bertoldi et al. (2008) are produced through the application of a magnetic field instead of a compressive force. The magneto-elastic lattice in Tipton et al. (2012) requires the application of a magnetic field to remain in the transformed state, and does not directly exploit the multistable nature of magneto-elastic lattices. This is the main objective of the present paper, which investigates how multistability associated with magneto-elastic interactions can be exploited for affecting the wave propagation properties of 1D and 2D magneto-elastic lattices.

The paper is organized as follows. Following this introduction, a general description of periodic lattices and a phenomenological modeling approach is discussed in Section 2. The approach leads to a relatively simple, lumped parameter framework that lends itself to a series of parametric investigations conducted herein. Next the methodology for the study of periodic lattices is discussed in Section 3. This includes the identification of equilibrium configurations (Section 3.2), and the determination of wave propagation using Bloch wave analysis (Section 3.3). Then, results for 1D and 2D structures, presented in Sections 4 and 5 respectively, show the effects that topological changes and lattice magnetization changes have on the propagation of waves. Finally, Section 6 summarizes the main findings of this study and provides recommendations for future investigations.

2. Theoretical background

2.1. Lattice configuration

Magneto-elastic lattices are modeled as systems of permanent magnetic particles with translational and rotational degrees of freedom. Axial and torsional springs connect the particles to form an elastic structure. The particles have a finite radius r_p, and feature both translational and rotational inertias (Fig. 1(a)). All particles are identical in terms of their inertial and magnetic properties. Similarly, all springs connecting the particles are massless and feature the same axial and torsional stiffness. The interactions between particles are governed by: (1) magnetic interactions, (2) elastic interactions through axial and torsional springs, and (3) axial and torsional viscous damping interactions. The energy functionals associated with the interactions above and the particle's degrees of freedom (DOF) are described in the following section.

2.2. Energy functionals

The behavior of the generic ith particle of a 2D lattice is described by the position of its center of mass $\boldsymbol{r}_i = x_i\boldsymbol{i} + y_i\boldsymbol{j}$, and by a rotation angle θ_i (Fig. 1(b)). Accordingly, the vector of the generalized coordinates associated with the ith particle is given by:

$$\boldsymbol{q}_i = [x_i,\ y_i,\ \theta_i]^T \tag{1}$$

The kinetic energy of each particle is expressed as:

$$T_i = \frac{1}{2}m(\dot{x}_i^2 + \dot{y}_i^2) + \frac{1}{2}I\dot{\theta}_i^2 \tag{2}$$

where $I = \frac{1}{2}mr_p^2$.

2.2.1. Mechanical interactions

The strain energy associated with the elastic interactions is defined in terms of axial and torsional contributions. A phenome-

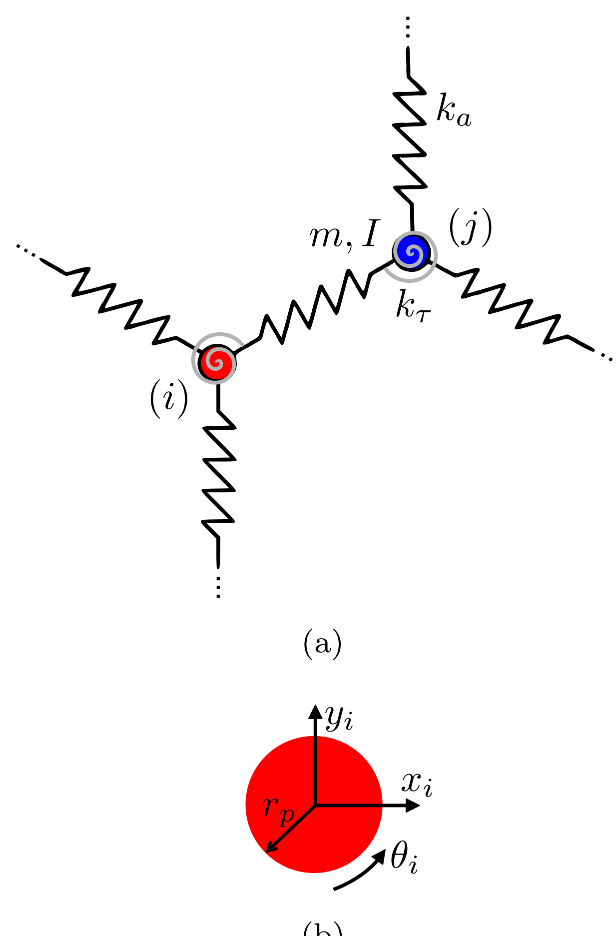

Fig. 1. Schematic of mechanical connectivity between two adjacent particles (a), and particle degrees of freedom (b).