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Stress distributions for blunt cracks and radiused slits in anisotropic plates under in-plane loadings

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ABSTRACT

An analytical solution for the in-plane stress fields in composite anisotropic plates with blunt cracks is derived using Lekhnitskii's approach. The orthotropic behaviour is obtained as a special case of the more general anisotropic solution.

Theoretical predictions are compared to results from a bulk of finite element analyses carried out on tensioned plates with parabolic and U-shaped notches, showing a very satisfactory agreement.

1. Introduction

The knowledge of the linear elastic stress fields ahead of cracks and notches is essential in the design of structural components, engineering strength criteria being almost based on quantities which can be directly correlated to local stress distributions (see, among the others, Berto and Lazzarin, 2014; Carraro et al., 2013; Carraro and Quaresimin, 2014 and references reported therein).

In the presence of a blunt notch the singularity of the linear elastic crack or pointed V-notch stress fields disappears and an asymptotical behaviour is evident only at a certain distance from the radiused notch tip. This stress redistribution, due to the finite value of the root radius ρ, makes the analytical solution for the notch stress fields much more complex than that associated to the corresponding sharp case, and exact analytical treatments are possible only for few "special" notch shapes.

With reference to isotropic plates, great efforts have been devoted in the recent years to enrich Williams' pioneering analysis for pointed V notches (Williams, 1952) by including the effect of a finite value of the notch root radius. Analytical solutions for the Mode I, II and III elastic stress fields in the vicinity of the tip of blunt cracks, or '*slim*' parabolic notches, in isotropic materials were derived by Creager and Paris (1967), the intensities of the fields being expressed as functions of Stress Intensity Factors. Using Neuber's conformal mapping (Neuber, 1958) and considering Mode I and Mode II loadings, Lazzarin and Tovo (1996) provided a linear elastic unifying solution capable of addressing any combination of notch tip radius and opening angle. This solution was later refined by Filippi et al. (2002) and Lazzarin et al. (2011). The linear elastic Mode III problem has been later addressed by Zappalorto et al. (2008, 2011) and Zappalorto and Lazzarin (2011a,b), who also dealt with the extension to the elastic–plastic case (Zappalorto and Lazzarin, 2009,2010).

Moreover, several efficient methods to determine the Notch Stress Intensity Factor at the V-notch tip have been developed in the recent years (see Lazzarin et al., 2008, 2010; Treifi and Oyadiji, 2013; Shi and Lu, 2013, and references reported therein).

Moving to anisotropic plates comparatively few works can be found in the literature, mainly oriented to refine the classical analysis for anisotropic plates with elliptical holes by Lekhnitskii (1984) and to make it applicable to composite laminates (see, among the others, Bonora et al., 1993, 1994; Chern and Tuttle, 2000). This interest is motivated by the fact that, independently of the far applied loads, the stress state close to a geometrical variation, such as a hole or a notch, is inherently multiaxial and under such a stress state the fatigue behaviour of composite materials might be very complex (see Quaresimin and Carraro, 2013, 2014, and Quaresimin et al., 2014 and references reported therein).

The stress distributions in an orthotropic plate with triangular holes were studied by Ukadgaonker and Rao (1999), while the case of an irregular shaped hole was later considered by Ukadgaonker and Rao (2000) and Ukadgaonker and Kakhandki (2005), where an excellent literature review on the topic can be found, as well.

A new three-dimensional theory to be applied to thick anisotropic plates with sharp V-notches was developed by the present authors (Zappalorto and Carraro, 2014), who also provided an

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engineering formula for the stress concentration factors of notched orthotropic plates (Zappalorto and Carraro, 2015).

The problem of a blunt crack in an anisotropic plate was addressed by Chiang (1994), with the main aim to determine the relationship between the maximum notch tip stress and the stress intensity factor of the corresponding sharp crack case.

The main aim of the present work is to analyse the rectilinearly anisotropic blunt crack problem by providing an exact solution for the stress fields close to the tip of a parabolic notch in a semi-infinite plate under in-plane loading conditions. The solution is obtained by using Lekhnitskii's complex approach and stresses are written as a function of two unknown constants which depend on the actual notch geometry and the far applied loading conditions. The orthotropic case is obtained as a particular case of the more general solution and, in this last mentioned case, uncoupled solutions are provided for symmetric in-plane loadings (tension or bending) and skew-symmetric in-plane loadings (in-plane shear). The accuracy of the solution is checked by comparison to a bulk of FE analyses carried out on finite size anisotropic and orthotropic plates weakened by parabolic notches. Eventually, the capability of the new developed solution to describe the stress fields due to U-shaped notches is verified as well.

2. Analytical preliminaries

In this work the material is supposed to have a plane of material symmetry which coincides with the plane of reference for the deformation field (rectilinear anisotropy). Accordingly, the elastic stress–strain relationships can be formulated on the basis of six independent elastic constants:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} \tag{1a}$$

for plane stress and:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} \tag{1b}$$

for plane strain, respectively. Constants B_{ij} can be expressed in terms of the compliances S_{ij}:

$$\begin{aligned} B_{11} &= \frac{S_{11}\,S_{33} - S_{13}^2}{S_{33}} \quad & B_{12} &= \frac{S_{12}\,S_{33} - S_{13}\,S_{23}}{S_{33}} \\ B_{22} &= \frac{S_{22}\,S_{33} - S_{23}^2}{S_{33}} \quad & B_{16} &= \frac{S_{16}\,S_{33} - S_{13}\,S_{36}}{S_{33}} \\ B_{66} &= \frac{S_{66}\,S_{33} - S_{36}^2}{S_{33}} \quad & B_{26} &= \frac{S_{26}\,S_{33} - S_{23}\,S_{36}}{S_{33}} \end{aligned} \tag{2}$$

Stress fields in the considered anisotropic body can be written in terms of two complex functions (Lekhnitskii, 1984):

$$\begin{aligned} \sigma_{xx} &= \mathrm{Re}\{\mu_1^2\phi_1(z_1) + \mu_2^2\phi_2(z_2)\} \\ \sigma_{yy} &= \mathrm{Re}\{\phi_1(z_1) + \phi_2(z_2)\} \\ \tau_{xy} &= -\mathrm{Re}\{\mu_1\phi_1(z_1) + \mu_2\phi_2(z_2)\} \end{aligned} \tag{3}$$

where μ_1 and μ_2 are unequal complex numbers defined as:

$$\mu_1 = \alpha_1 + i\beta_1 \quad \mu_2 = \alpha_2 + i\beta_2 \quad (\beta_1, \beta_2 > 0) \tag{4}$$

and represent the roots of the following characteristic equations:

$$S_{11}\mu^4 - 2S_{16}\mu^3 + (2S_{12} + S_{66})\mu^2 - 2S_{26}\mu + S_{22} = 0 \tag{5}$$

for plane stress and

$$B_{11}\mu^4 - 2B_{16}\mu^3 + (2B_{12} + B_{66})\mu^2 - 2B_{26}\mu + B_{22} = 0 \tag{6}$$

for plane strain

3. Mathematical description of the notch profile

Notch tip rounding is described by using a parabolic notch profile. Consider the orthogonal curvilinear coordinate system generated by the following transformation (Neuber, 1958):

$$z = x + iy = (u + iv)^2 = w^2 \tag{7}$$

where z and w are complex variables in the physical and the mapped plane, respectively.

The curvilinear coordinate system (u, v) here introduced allows one to completely describe a parabolic profile (see Fig. 1). The generic curve characterised by the coordinate u_0 intersects the x-axis at a value:

$$r_0 = \frac{\rho}{2} = u_0^2 \tag{8}$$

where ρ is the curvature radius at the notch tip. The notch edge equation, instead, reads as:

$$r(\theta) = \frac{\rho}{2}\cos^{-2}\frac{\theta}{2} \tag{9}$$

4. Stress distributions for anisotropic plates

Consider the following complex functions:

$$\begin{aligned} \phi_1(z_1) &= A\frac{\mu_2}{\mu_1 - \mu_2}z_1^{-\frac{1}{2}} + B\frac{1}{\mu_1 - \mu_2}z_1^{-\frac{1}{2}} \\ \phi_2(z_2) &= -A\frac{\mu_1}{\mu_1 - \mu_2}z_2^{-\frac{1}{2}} + B\frac{1}{\mu_1 - \mu_2}z_2^{-\frac{1}{2}} \end{aligned} \tag{10}$$

In Eq. (10) A and B are real quantities, while z_j are complex variables defined as:

$$z_j = \xi_j + i\eta_j = r_j e^{i\theta_j} \tag{11}$$

where:

$$\begin{aligned} \xi_j &= x' - \frac{\rho}{2}\left(\alpha_j^2 - \beta_j^2\right) + \alpha_j y' \\ \eta_j &= \beta_j y' - \rho\alpha_j\beta_j \end{aligned} \tag{12}$$

Moreover:

$$\begin{aligned} r_j &= \sqrt{\xi_j^2 + \eta_j^2} \\ \theta_j &= \mathrm{Arg}\left(\xi_j + i\eta_j\right) \end{aligned} \tag{13}$$

In Eqs. (11)–(13), x' and y' are the distances from the notch tip in the x and y directions, respectively (see Fig. 1).

Stress components can be determined by substituting Eqs. (10) into Eqs. (3). After some algebraic arrangements one obtains:

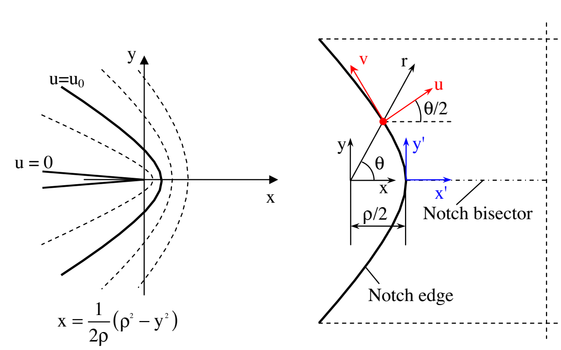

Fig. 1. Description of a notch with a parabolic profile.