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Variationally consistent derivation of the stress partitioning law in saturated porous media

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ABSTRACT

The linearized version of a recently formulated Variational Macroscopic Theory of biphasic isotropic Porous Media (VMTPM) is employed to derive a general stress partitioning law for media undergoing flow conditions under prevented fluid seepage and negligible inertia effects, typically met in biphasic specimens subjected to jacketed tests.

The principle of virtual work, relevant to the specialization of VMTPM to such characteristic flow conditions, naturally yields a stress partitioning law, between solid and fluid phase of a saturated medium, that exactly matches with the celebrated Terzaghi's principle. It is also shown that the stress tensor of the solid phase work-associated with the strain measure of the VMTPM naturally corresponds to the Terzaghi's effective stress. Accordingly, under undrained conditions, Terzaghi's law is proved to be a completely general stress partitioning law for a saturated biphasic medium irrespective of its constitutive and/or microstructural features as well as of the compressibility of its constituent phases.

Since the developments reported are obtained ruling out thermodynamic constraints and any assumption on the internal microstructure and on the compressibility of the phases, the results obtained indicate that Terzaghi's law could more generally apply to a broader class of biphasic media and be not restricted within the context of geomechanics.

1. Introduction

Understanding how an externally applied stress is partitioned among the constituent phases of multiphase porous media represents a challenging problem in solids mechanics. This issue has received much attention both from a purely theoretical point of view, as a general problem of continuum mechanics, (Bedford and Drumheller, 1983; Bennethum, 2006; Bennethum and Cushman, 1996; Bowen, 1982; Coussy et al., 1998; De Boer, 1996; Ehlers, 2002; Gajo, 2010; Gray et al., 2009; Truesdell, 1957), and in more applicative contexts, such as soil mechanics (Skempton, 1960a; dell'Isola and Hutter, 1998), biomechanics (Ateshian et al., 1997; Lai and Mow, 1980; Mow et al., 1980) and impact engineering (Markert, 2008), dealing with specific classes of multiphase media.

Even restricting attention to the simpler subclass of completely saturated two-phase porous media, it can be recognized that a generally agreed solution for the stress partitioning problem in generic multiphase media is still not available. This conclusion is reached by scoping both the results available from theoretical research and those coming from experimental applied research.

When proceeding from a theoretical approach, research has necessarily focused on endowing the notions of total stress and partial phase stresses with precise, mechanically-consistent (Bedford and Drumheller, 1979; Hassanizadeh and Gray, 1979) and thermodynamically admissible (Gray et al., 2009; Svendsen and Hutter, 1995) mechanical definitions, as well as on the deduction of the stress partitioning law in the context of the many existing rational continuum theories of immiscible mixtures. In this respect, a multiplicity of immiscible continuum theories of mixtures has been proposed, see, e.g., the many works referenced in surveys (Baveye, 2013; Bedford and Drumheller, 1983; Gray et al., 2013), although these do not often collimate neither in the macroscopic balance equations (Albers and Wilmanski, 2006; Biot, 1955; Bowen, 1982; Goodman and Cowin, 1972; Hassanizadeh and Gray, 1979; Wilmanski, 1998), nor for the very physical–mathematical, or engineering, definition considered to introduce macroscopic stress measures in the theory (Bedford and Drumheller, 1979; Biot and Willis, 1957; Bishop, 1959;

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Coussy et al., 1998; de Boer and Ehlers, 1990; de Boer, 2005; Gray and Hassanizadeh, 1991; Gray et al., 2009; Lancellotta, 2002; Nur and Byerlee, 1971; Schrefler, 2002; Skempton, 1960a; Wilmanski, 2004).

Similarly, when proceeding from an experimental point of view, non-exhaustive answers are found when trying to extend, to laws of more broad applicability, the stress partitioning regime measured for a particular class of biphasic media. Actually, a particular regime of stress partitioning measured for a given tested medium may be intrinsically affected by some of its specific microstructural features, mechanical properties of its constituent phases and by the flow condition determined by the particular experimental set-up employed. These generic statements are better clarified by their contextualization for the stress partitioning problem in the mechanics of saturated soils and rocks.

Due to the relevance of the consolidation problem in Hydrology and Geotechnics, soil and rock mechanics have been historically primary sources of experimental and theoretical results which have significantly contributed to the advancement in understanding the mechanics of stress partitioning in biphasic media (de Boer, 1998). Moreover, the variability of the geomaterials retrievable in nature has provided a favorable ground to analyze stress partitioning for media with very differentiated microstructures and volumetric fractions, ranging from granular materials (soils) to cemented materials, thus providing a valuable database to approach the stress partitioning problem in a general sense comprehensive of a broad class of media.

Nevertheless, even restricting attention to the simpler (binary) problem of saturated rocks and soils, it can be recognized that a unique accepted stress partitioning law of general applicability to the biphasic media encountered in geomechanics is not currently available. An anchorage of unquestionable evidence in geomechanics is the validity of Terzaghi's effective stress law (Terzaghi, 1936) in describing stress partitioning in saturated soils. This law has been carefully assessed for saturated soils, rocks and concrete subjected to different jacketed, unjacketed and triaxial test conditions, (Skempton, 1960a), and is known to provide an accurate and well established *experimental true form* of stress partitioning for saturated granular soils, up to be celebrated as 'Terzaghi's principle' in Geotechnics.

Assuming normal tensile stresses and compressive pressures as positive, this law can be written in intrinsic tensor form as follows

$$\boldsymbol{\sigma}^{(s)} = \boldsymbol{\sigma} + p\boldsymbol{I}, \tag{1}$$

where $\boldsymbol{\sigma}^{(s)}$ is Terzaghi's effective stress of the solid phase, $\boldsymbol{\sigma}$ is the total stress, and p the fluid pressure.

However, confidence in the applicability of relation (1) for saturated soils provides only a partial solution to the stress partitioning problems of interest of Geomechanics, since saturated granular materials are not the unique biphasic media encountered in geotechnical applications. The range of applicability of (1) has been experimentally explored, beyond granular materials, for other classes of biphasic geomaterials such as saturated rocks and concrete. A review of the proposed representations of the effective stress for saturated and unsaturated soils has been provided by Jardine et al. (2004) and Nuth and Laloui (2008). As reported in the quoted papers, it is generally believed that relation (1) is not sufficiently general to describe the mechanics of stress partitioning for generic biphasic fully saturated geomaterials. To achieve a more general stress partitioning law comprehensive of saturated rocks, most of the laws so far proposed for stress partitioning consider a modification of Terzaghi's law whereby (1) is recovered for soils only as a special case. These can be all framed into the general representation:

$$\boldsymbol{\sigma}^{(s)} = \boldsymbol{\sigma} + \alpha p\boldsymbol{I}, \tag{2}$$

where the expression and the meaning of the coefficient α vary according to the biphasic framework adopted by the different authors. The conclusion of both Jardine and co-workers and Nuth and Laloui is that there is no available expression for α valid for all classes of soil and rock materials. Most of the expressions proposed for α address a dependency of this parameter upon the ratio $\frac{C_s}{C}$, where C_s is the intrinsic compressibility of the grain and C is the macroscopic compressibility of the porous skeleton, see Table 1. For all the expressions in Table 1, Terzaghi's law for soils (1) is recovered as the ratio $\frac{C_s}{C}$ vanishes, what corresponds to the condition that the macroscopic compressibility of the porous skeleton is much higher than the intrinsic compressibility of the grain. This modality of recovering (2) reflects the prevailing current opinion in Geomechanics which connects the validity of Terzaghi's law with the presence of a low ratio $\frac{C_s}{C}$. Most of the currently widespread porous media models agree with the expression $\alpha = 1 - \frac{C_s}{C}$, even if they proceed from different theoretical frameworks and from different originary definitions for α. For instance, in 1955 Biot proposed a model where $\alpha = \phi_o^{(f)}$ (the porosity) (Biot, 1955). Nevertheless, with reference to his deformation theory of poroelasticity, the same author presented two years later (Biot and Willis, 1957) methods of measurement of the elastic coefficients in agreement with $\alpha = 1 - \frac{C_s}{C}$. A similar agreement is found in Gray and Schrefler (2007) where, employing methods of averaging of microscale conservation equations, it is shown that this stress partitioning coefficient corresponds to the ratio of the hydrostatic part of the total stress tensor to the hydrostatic part of the solid phase stress tensor.

Even framing back Terzaghi's law in a more theoretical perspective, unquestionable agreement is not found on the mechanical significance of relation (1), i.e., on which is the peculiar mechanical condition met in saturated granular materials which makes the stress to be partitioned in compliance with relation (1). When both solid and fluid phases are assumed to be incompressible, a rational justification of the recovery of Eq. (1) has been provided by de Boer and Ehlers (1990) by showing that an effective stress relation taking a form comparable to (1) is obtained from considerations involving thermodynamic constraints with the employment of Lagrange multipliers.

On the other hand, the existence of a mandatory connection between an incompressibility hypothesis and the recovery of Terzaghi's law is not unanimously agreed. For instance, further partitioning laws that resemble the Terzaghi's stress concept have been reported by Hassanizadeh and Gray (1990) and Schrefler (2002). In these studies, based on averaging approach and considerations stemming from the entropy inequality, the recovery of Terzaghi's law is brought in connection with a thermodynamic near-equilibrium condition rather than with a property of low intrinsic compressibility of phases, thus raising questions on the strict necessity of the incompressibility hypothesis at the base of relation (1).

Table 1
Expression for α in the effective stress equation (2), extracted from (Jardine et al. (2004).

Expression for α	Reference
$\phi_o^{(f)}$	Biot (1955)
1	Terzaghi (1936)
$1 - \frac{C_s}{C}$	Skempton (1960a) Biot and Willis (1957) Nur and Byerlee (1971) Bishop (1973) Gray and Schrefler (2007)
$1 - (1 - \phi_o^{(f)})\frac{C_s}{C}$	Lade and De Boer (1997) Suklje (1969)