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On the almost-complete contact of elastic rough surfaces: The removal of tensile patches

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ABSTRACT

The "almost-complete" contact of elastic rough surfaces is studied by the method introduced by Johnson of reducing the mean pressure below the value needed for full contact, and then removing the resulting patches of tensile stresses by superposition (Johnson et al., 1985). The procedure is examined for circular tensile patches using Sneddon's (1946) equations, applied to a variety of (tensile) pressure distributions: Sneddon's analysis is extended slightly to find the associated pressure distributions. It is found that the removal of circular patches of tensile stress does not alter the total load. The total out-of-contact area is always greater than the total area of tensile stress, but the increase depends on the behaviour of the pressures adjacent to the tensile region, and area increases of any amount from 14% upwards can be obtained with plausible pressure distributions: the hope that the increase will always be close to 50% is unwarranted.

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1. Introduction

Since the Bowden & Tabor theory, that the area of contact between rough surfaces is determined by a 'flow pressure' resembling the hardness, was challenged by Archard and his colleagues at the AEI Laboratories, there have been many attempts, beginning with Archard's own, to explain the proportionality between the area of contact and the load while still assuming elastic behaviour. And indeed, we now understand the principle enunciated by Archard: that if the effect of increasing the load is to make existing contacts grow, there will not be proportionality: but if the effect is primarily to form new contacts, there may well be. Attempts to quantify this by, among others, Greenwood and Williamson (1966), Whitehouse and Archard (1970), and Bush et al. (1975), are well-known: but all sit in the Bowden & Tabor scenario of the real area of contact being a very small fraction of the apparent area: in other words, of the *beginnings* of contact as the surfaces first approach each other.

More recently there have been attempts to understand the *last* stages of contact: when there is contact almost everywhere (Xu et al., 2014; Yastrebov, 2014a,b). These last stages of contact are relevant to the amount of trapped lubricant, particularly important for tyre/road contacts (but the same problem in foil-rolling is not a problem of *elastic* contact!). It has also been suggested that the final stages of contact are related to leakage through seals, though leakage is perhaps more directly linked to the percolation limit. There is also the natural wish to obtain a complete picture of the contact process. But possibly the most important aspect is its relation to Persson's widely used theory of contact, in which partial contact is obtained as a modification of perfect, complete, contact – although by a startlingly different method from that to be considered here (e.g. Persson, 2002).

It is well known that when two elastic, rough, surfaces are loaded together, complete contact can be achieved with a pressure distribution related to the surface roughness through its spectral density (provided, of course, that the materials obey the linear elastic equations at all strains!). The simplest example is that a periodic surface roughness $z = a\sin(kx)$ can be completely flattened by pressures $p = a(E'k/2)\sin(kx)$ where E' is the plane strain modulus $E/(1-\nu^2)$: this can readily be generalised (see Johnson et al. (1985) or Persson (2002). [We consider here the contact of a rough elastic half-space with a rigid plane: the generalisation to two elastic half-spaces is straightforward (see Johnson (1985)).] In the first instance there will be both compressive and *tensile* normal stresses over the interface, but a uniform pressure can be superposed without affecting the contact, and if sufficiently large, will suppress all the tensile regions.

The analysis of incomplete contact starts from here, but the uniform pressure giving this state of complete contact is slightly reduced, so that small, isolated, tensile regions will appear. In the absence of adhesion, this is impossible; and the true pressure

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distribution must be obtained by removing these tensile regions. This paper discusses the removal procedure in detail.

2. Removal of circular patches of tensile stresses

Following Johnson et al. (1985) and Xu et al. (2014), it is reasonable to approximate the tensile stresses over an individual "contact" as paraboloidal over a circular area of radius a, and so of the form $\sigma = -p_0(1 - r^2/a^2)$. Then applying pressures $p = +p_0(1 - r^2/a^2)$ would eliminate the tensile region. But imposing such pressures *alone* would produce deformation *outside* the circular patch, and so would disturb the contact outside the circle: instead it is necessary to use a pressure distribution which eliminates the tensile stresses and has no effect over the remainder of the surface: that is, to use the solution from fracture mechanics for pressures applied to a penny-shaped crack.

However, if this solution is applied for a penny-shaped crack of the same radius $r = a$, we obtain infinite stresses around the periphery of the crack. To avoid these, the additional stresses must be applied over a larger area $r > a$: so that not only are the tensile stresses removed but also some of the surrounding compressive stresses. Explicitly, the stress intensity factor for stresses applied over a circle $r \leqslant c$ is $K_I = \frac{2}{\sqrt{\pi c}} \int_0^c \frac{r\,p(r)}{\sqrt{c^2-r^2}} dr$: and clearly this can only vanish if $p(r)$ changes sign within $r \leqslant c$. In particular, if $p = p_0(1 - r^2/a^2)$ is applied over a larger circle $r \leqslant c$, (pressures over $r < a$, but additional *tensile* stresses over $a < r < c$!), then $K_I = \frac{2cp_0}{\sqrt{\pi c}}[1 - (2/3)c^2/a^2]$, and to make K_I vanish the pressures must be applied over a circle of radius $c = a\sqrt{3/2}$. Manners and Greenwood (2006) note that the corresponding increase in width for a line contact is $b = x_0\sqrt{2}$, so that in both cases the *area* increase is by a similar factor: and they suggest that this factor may hold generally.

Now, for the *regular* wavy surfaces studied by Johnson et al., the approximation of parabolic tensile stresses is plausible: but is it reasonable for the last stages of contact of the fractal surfaces studied by Persson, or the Gaussian rough surfaces studied by Xu et al. (2014)? Indeed, this work was prompted by the suggestion by Xu et al. that the tensile pressure may have been over-simplified as parabolic.

Accordingly, we investigate below in more detail the procedure for removing tensile pressures: restricting attention to circular regions for which we can rely on the results in Sneddon's classic treatise (Sneddon, 1946).

3. Theory

As Johnson explains, since the surfaces are considered to be in contact everywhere with the aid of the tensile stresses, it is necessary to remove the tensile stresses *without* affecting the contact elsewhere: a solution is needed for the ***mixed*** boundary value problem of given stresses over the circle $r < c$ but zero displacement over the region $r > c$. A summary of Sneddon's analysis, and of the extension to find the stresses, is given in the Appendix A: here we quote the final results: that a pressure[1] $p(r) = q_n(r/c)^n$ over $r < c$ can only produce zero displacements over $r > c$ when associated with stresses

$$\sigma_z|_{z=0} = +\frac{1}{\sqrt{\pi}} q_n \frac{(n/2)!}{((n+1)/2)!}\left[(r^2/c^2-1)^{-1/2} - (n+1)J_n\right] \quad \text{over } r > c,$$

where $nJ_n = (n-1)(r/c)^2 J_{n-2} - (r^2/c^2 - 1)^{1/2}$ for $n > 1$ and $J_0 = \sin^{-1}(c/r)$.

[1] Coefficients q_n will be used for terms $(r/c)^n$, reserving p_n for $(r/a)^n$.

The displacements within $r < c$ will then be

$$w(r) = \frac{2c}{\sqrt{\pi}} \frac{q_n}{E'} \frac{(n/2)!}{((n+1)/2)!} I_n(r/c), \quad \text{over } r < c,$$

where $(n+1)I_n = (1 - r^2/c^2)^{1/2} + n(r/c)^2 I_{n-2}$ and $I_0 = (1 - r^2/c^2)^{1/2}$.

Thus, writing $\rho = r/c$, a uniform pressure q_0 must be associated with

$$\sigma_z|_{z=0} = +\frac{2q_0}{\pi}\left[(\rho^2-1)^{-1/2} - \sin^{-1}\frac{1}{\rho}\right] \qquad \text{(Sneddon 3.5.3)}$$

and will give $w_0(r) = \frac{4}{\pi}\frac{q_0}{E'}\sqrt{c^2 - r^2}$, while a pressure $q_2(r/c)^2$ must be associated with

$$\sigma_z|_{z=0} = +\frac{2q_2}{\pi}\left[(2/3)(\rho^2-1)^{-1/2} + (\rho^2-1)^{1/2} - \rho^2 \sin^{-1} 1/\rho\right]$$

and will give $w_2(r) = \frac{8}{9\pi}\frac{q_2}{E'}(1 + 2(r/c)^2)\sqrt{c^2 - r^2}$.

For a general pressure distribution $p = \sum_{n \text{ even}} p_n(r/a)^n$, the singular terms give (as $r \to c$) $\sigma_z|_{z=0} = +\frac{1}{\sqrt{\pi}}(r^2/c^2 - 1)^{-1/2} \sum_{n \text{ even}} \frac{(n/2)!}{((n+1)/2)!} p_n(c/a)^n$, so for the stresses to remain finite the sum of the series must vanish: that is, we must have $p_0 + \frac{2}{3}p_2(c/a)^2 + \frac{2.4}{3.5}p_4(c/a)^4 + \frac{2.4.6}{3.5.7}p_6(c/a)^6 + \frac{2.4.6.8}{3.5.7.9}p_8(c/a)^8 + \cdots = 0$. This equation must be solved to find c/a. (The same equation is obtained by setting the SIF integral to zero: $K_I = \frac{2}{\sqrt{\pi c}}\int_0^c \frac{r\,p(r)}{\sqrt{c^2-r^2}} dr = 0$. This does not rely on Sneddon's detailed analysis, so is not restricted to polynomials: see below.)

3.1. Johnson's method

As an alternative to requiring the stresses to remain finite, Johnson et al. (1985) find the condition for the slope at the contact edge to be zero, so that the surfaces separate smoothly. From Sneddon's equations (as quoted above), they note that a pressure distribution $p = p_0(1 - r^2/a^2) \equiv q_0 - q_2(r^2/c^2)$ will give displacements

$$w(r) = \frac{4}{\pi}\frac{q_0}{E'}\sqrt{c^2 - r^2} - \frac{8}{9\pi}\frac{q_2}{E'}(1 + 2(r/c)^2)\sqrt{c^2 - r^2}$$

so that $\frac{\partial w}{\partial r}\big|_{r=c}$ will be infinite unless $\frac{4}{\pi}\frac{q_0}{E'} - \frac{8}{9\pi}\frac{q_2}{E'}(1 + 2) = 0$: i.e. unless $q_2 = \frac{3}{2}q_0$ as before. It will be seen that the requirement of a *finite* slope at the contact edge results in the slope there becoming zero.

(Xu et al. (2014) also verify that for parabolic pressures, the two methods give the same result.)

4. Results

Fig. 1 shows the results for the parabolic stress distribution considered by others: the green curve is the assumed initial distribution, with both tensile and compressive parts: the red curve represents the pressures needed to give zero pressures over $r \leqslant c$, and the blue curves are the associated pressures needed to preserve contact in the region surrounding the patch of no contact. Note that although *initially* there is a pressure $+p_0/2$ at $r = c$, and that *now* there are zero pressures over $r < c$, there is in fact no discontinuity: the magenta (dash-dot) curve is the final pressure distribution, showing that the pressures are continuous at $r = c$, although very quickly returning to the curve of the original parabolic stresses. Explicitly, the additional stresses are

$$\sigma_z|_{z=0} = +\frac{2p_0}{\pi}\left[-\sin^{-1}\frac{1}{\rho} + \frac{3}{2}\left\{\rho^2 \sin^{-1} 1/\rho - (\rho^2 - 1)^{1/2}\right\}\right].$$

Note also (see Appendix A) that the additional pressures are self-equilibrating, so that the load bringing the surfaces together is unchanged by the elimination of the tensile regions. The suggestion by Xu et al. that a small error in nominal pressure will occur is incorrect.

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