International Journal of Solids and Structures 52 (2015) 56–71

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Green's functions and extended displacement discontinuity method for interfacial cracks in three-dimensional transversely isotropic magneto-electro-elastic bi-materials

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ARTICLE INFO

Article history:
Received 12 March 2014
Received in revised form 18 September 2014
Available online 2 October 2014

Keywords:
Magneto-electro-elastic bi-materials
Interfacial crack
Extended displacement discontinuity method
Electrically and magnetically impermeable
Gaussian distribution function

ABSTRACT

In this paper, we derive the Green's functions of constant extended interfacial displacement discontinuities within a rectangular element and of point extended interfacial displacement discontinuities in three-dimensional transversely isotropic magneto-electro-elastic (MEE) bi-materials. The derived Green's functions along with the extended displacement discontinuity method are applied to analyze the electrically and magnetically impermeable interfacial cracks in the three-dimensional MEE bi-materials. To deal with the oscillatory singularities at the crack front, the Dirac delta function in the Green's functions is replaced by the Gaussian distribution function, and correspondingly, the unit Heaviside function is approximated by the Error function. Numerical study illustrates the effect of the ε parameter in the Gaussian distribution function on the *J*-integral. The stress intensity factors, electric displacement intensity factor, and magnetic induction intensity factor are expressed in terms of the extended displacement discontinuities. The influence of different MEE material mismatches as well as different extended loadings (uniformly or non-uniformly distributed on the crack face) on the fracture parameters is investigated. Different rectangular crack sizes are also considered in the numerical simulation.

1. Introduction

Since Williams (1959) first investigated a semi-infinite interfacial crack in elastic dissimilar media, increasing enthusiasm follows in studying the behaviors and characters of interfacial cracks. The oscillatory singularity at the two-dimensional (2D) interfacial crack tip was discussed (Erdogan, 1965; England, 1965; Rice and Sih, 1965; Suo and Hutchinson, 1990) and different approaches were proposed to deal with this unsatisfactory oscillatory behavior (e.g., Atkinson, 1977; Comninou, 1977, 1990; Dundurs and Gautesen, 1988). Related works on the corresponding three-dimensional (3D) interfacial fracture problems can be found in Willis (1971), Lazarus and Leblond (1998a,b), Antipov (1999), Bercial-Veleza et al. (2005) and Pindra et al. (2008), among others. One efficient way of simulating interfacial cracks is by interfacial dislocations (Eshelby, 1951; Comninou, 1977; Qu and Li, 1991). But the oscillating singularity would still exist with the Dirac delta function in the solutions. Mathematically, it has been proved that a variety of appropriate approximations of the Dirac delta function can be made depending on the specific physical or engineering problems under consideration. Zhang and Wang (2013) reconsidered the dislocation approach for interfacial cracks and replaced the Dirac delta function with the Gaussian distribution function to eliminate the oscillatory singularity.

The analysis of the elastic interfacial cracks was then extended to the piezoelectric bi-material (Kuo and Barnett, 1991; Suo et al., 1992; Beom and Atluri, 1996; Qin and Mai, 1999; Herrmann and Loboda, 2000; Zhao et al., 2008b). One of the most interesting findings from these studies was that besides the classical singularity $r^{-1/2}$ and the well-known oscillatory singularity $r^{-1/2\pm i\varepsilon}$, the extended stresses have a new type of singularity $r^{-1/2\pm\kappa}$ near the crack tip in 2D and also in 3D piezoelectric bi-materials. It was also found that an impermeable interfacial crack in the transversely isotropic bi-materials could be classified into two types according to the feature of the ε-singularity and κ-singularity of the stress field near the crack tip.

The first magneto-electro-elastic (MEE) composite was fabricated by Van Run et al. (1974). In recent two decades, due to the interesting coupling features among the mechanical, electrical

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http://dx.doi.org/10.1016/j.ijsolstr.2014.09.018
0020-7683/

and magnetic fields, MEE composites have attracted extensive attentions from different branches of science and engineering. Since defects may exist in these novel composites, fracture problems in them need to be addressed. Representative contributions in this direction include 2D in-plane fracture problems (Song and Sih, 2003; Sih and Song, 2003; Gao et al., 2003; Wang and Mai, 2003) and anti-plane problems (Wang and Shen, 1996; Spyropoulos et al., 2003; Gao et al., 2004; Chue and Liu, 2005; Zhong and Li, 2006; Hu and Chen, 2014).

For the study of the crack problems in MEE bi-materials, all the previous works, such as Gao et al. (2003), Zhou et al. (2004) and Tian and Gabbert 2005, helped shed light on the understanding of 2D interfacial fracture behaviors of MEE bi-materials. The study on the corresponding 3D interfacial fracture problems is more important in theory and practical in engineering applications. In this direction, Zhao et al. (2008a) analyzed the interfacial crack of an arbitrary shape in 3D transversely isotropic MEE bi-materials by extending their 3D piezoelectric bi-material approach (Zhao et al., 2008b). They found that as its counterpart in piezoelectric bi-materials, the stress near the crack front in MEE bi-material also has two kinds of singularity, namely, the oscillating and non-oscillating singularities, depending on the specific MEE bi-material system. They further explained that these two singularities cannot coexist in the same bi-material system. Zhu et al. (2009) adopted the integro-differential equation method to analyze the 3D interfacial crack in MEE bi-material. In their work, the unknown displacement discontinuities along the crack face were approximated by the product of extended basic density functions and polynomials and the resulting integro-differential equations were solved numerically.

The displacement discontinuity method was first proposed by Crouch (1976) to study the crack problems in elasticity numerically. Later studies showed that this method is efficient and flexible, and further can be conveniently applied to analyze 3D crack problems in piezoelectric media (Zhao et al., 1997) and in MEE materials (Zhao et al., 2007) by extending the original elastic displacement discontinuity to include the piezoelectric potential and magnetic potential.

In this paper, we first apply the boundary integro-differential method (i.e., Zhao et al., 2008a) to derive the Green's functions of the extended interfacial displacement discontinuities in 3D transversely isotropic MEE bi-material. The Dirac delta function in the Green's function is then approximated by the Gaussian distribution function as in Zhao et al. (2014) to remove the oscillatory singularities. Correspondingly, the approximation of the unit Heaviside function is introduced based on its relation with the Dirac delta function. Using the obtained 3D Green's functions, the extended displacement discontinuity method is applied to analyze the interfacial cracks in 3D MEE bi-materials.

This paper is organized as follows: Section 2 outlines the basic equations of the MEE material. In Section 3, solutions, especially those on the interface, for an interfacial crack in 3D MEE bi-material are obtained via the integro-differential approach which is extended from Zhao et al. (2008a). Then the Green's functions of the constant extended interfacial displacement discontinuities within a rectangular element and the point extended interfacial displacement discontinuities are derived. In Section 4, the Dirac delta function and the unit Heaviside function in the Green's functions are approximated, respectively, by the Gaussian distribution function and the Error function. The analytical expressions for the stress intensity factors, electric displacement intensity factor, magnetic induction intensity factor and the energy release rate are also given. Extended displacement discontinuity method is introduced in Section 5. In Section 6, numerical results are presented to illustrate the effect of the ε parameter in the Gaussian distribution function on the J-integral. The influence of different MEE material mismatches and different extended loadings (uniformly or non-uniformly distributed on the crack face) on the crack parameters is further investigated. Conclusion is drawn in Section 7.

2. Basic equations

In a three-dimensional Cartesian coordinate system x_i ($i = 1,2,3$), the governing equations for a linear transversely isotropic MEE medium without body force and free from electric charge and current are given by (1) the equilibrium equation, (2) the kinematic equation, and (3) the constitutive equation as listed below:

$$\sigma_{ij,j} = 0, \quad D_{i,i} = 0, \quad B_{i,i} = 0, \tag{1}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\varphi_{,i}, \quad H_i = -\psi_{,i}, \tag{2}$$

$$\begin{aligned} \sigma_{ij} &= c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - f_{kij}H_k, \\ D_i &= e_{ikl}\varepsilon_{kl} + \kappa_{il}E_l + g_{il}H_l, \\ B_i &= f_{ikl}\varepsilon_{kl} + g_{li}E_l + \mu_{il}H_l, \end{aligned} \tag{3}$$

where σ_{ij}, D_i and B_i are stress, electric displacement and magnetic induction components, respectively, and they are called extended stresses. u_i, φ and ψ are displacement components, electric potential and magnetic potential and are called extended displacements. ε_{ij}, E_i and H_i denote, respectively, strain components, electric field and magnetic field. c_{ijkl}, e_{ikl}, f_{ikl}, κ_{il}, g_{il} and μ_{il} are elastic constants, piezoelectric constants, piezomagnetic constants, dielectric permittivity, electromagnetic constants and magnetic permeability, respectively. A subscript comma denotes the partial differentiation with respect to the coordinate, with repeated indices taking their summation from 1 to 3 (Huang et al., 1998; Pan, 2002; Zhao and Fan, 2008).

3. Green's functions of extended interfacial displacement discontinuities

3.1. Integro-differential expressions of extended stresses due to an interfacial crack

We consider a transversely isotropic MEE bi-material system with its interface parallel to the plane of isotropy and lies on the ox_1x_2-plane, as schematically shown in Fig. 1. The poling direction is along the x_3-axis. A flat crack of arbitrary shape lies on the interface. The upper and lower faces of this interfacial crack are denoted, respectively, by S^+ and S^- with their outer normal vectors being

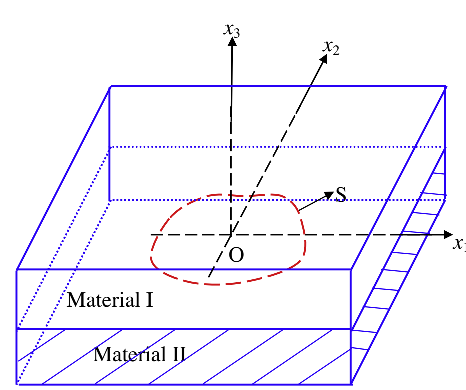

Fig. 1. An interfacial crack of an arbitrary shape under extended loadings on its surface in a transversely isotropic MEE bi-material.