International Journal of Solids and Structures 52 (2015) 72–82

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Two-dimensional transient analysis of wave propagation in functionally graded piezoelectric slabs using the transform method

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ARTICLE INFO

Article history:
Received 3 March 2014
Received in revised form 31 August 2014
Available online 6 October 2014

Keywords:
FGPM
Stress waves
Durbin method
Numerical Laplace inversion
Transient response

ABSTRACT

In this study, the transient responses of a functionally graded piezoelectric slab were analyzed using the transform technique. The slab was subjected to a dynamic anti-plane concentrated force and an in-plane point electric displacement on the top surface, and the bottom surface was assumed to be open-circuit or grounded. The analytical solutions were obtained in the Laplace transform domain, and a numerical inversion was performed using the Durbin method. The numerical results showed that an applied point electric displacement may instantaneously induce shear stress waves due to the piezoelectricity for both open-circuit and grounded cases. When the bottom surface was grounded, a visible electric wave was induced by the grounded boundary and the influence of this wave was significant when the gradient coefficient of the functionally graded piezoelectric materials (FGPMs) was large enough.

1. Introduction

Piezoelectric materials possess the important property of coupling between mechanical and electrical fields, which renders them useful in many areas of modern technology. These materials have therefore been widely used for a long time as electromechanical transducers, filters, sensors and actuators, to mention only a few applications. In recent years, various applications of piezoelectric materials have been implemented in non-destructive evaluation, ultrasonic medical imaging, smart structures, and the active control of sound and vibration. To increase the lifetime and reliability of advanced piezoelectric structures, functionally graded material has been considered as a way to improve interface problems. This new type of piezoelectric material is referred to as a functionally graded piezoelectric material. A functionally graded material (FGM) can be prepared by continuously changing the constituents of multi-phase materials in a predetermined volume as a fraction of the constituent material (Khor et al., 1997; Kwon and Crimp, 1997; Nogata, 1997). Most researchers have analyzed the composition of FGPMs with three types of functions, power-law, polynomial, and exponential, which are widely used because these functions provide advantages in theoretical investigation. For the power-law case, Wu et al. (2003) considered the material constants of a FGPM cylindrical shell as a power-law functions in the radial direction. When an axisymmetric thermal or mechanical loading is applied on the cylindrical shell, an exact solution is obtained through the power series expansion method together with the Fourier series expansion method. For the polynomial FGPM, Yu et al. (2007, 2009) used the Legendre orthogonal polynomial series expansion to determine the wave characteristics in spherically curved plates or hollow cylinders composed of FGPMs with an open-circuit condition. For the FGPM with an exponential function variation, Li et al. (2004) investigated the dispersion relations of Love waves in a layered functionally graded piezoelectric structure for electric open and short cases. Zhong and Shang (2003) presented an exact three-dimensional solution of the FGPM rectangular plate for the simply supported and grounded boundary. Zhong and Yu (2008) proposed a two-dimensional general solution for FGPM beams with arbitrary graded functions, and the numerical calculation was based on the cantilever beam with exponential variation. Time-harmonic response of a vertically graded transversely isotropic, linearly elastic half-space is analytically determined by Eskandari-Ghadi and Amiri-Hezaveh (2014) by introducing a new set of potential functions. The potential functions are set in such a way that the governing equations be simple and with physical meaning as well.

The study of wave propagation in piezoelectric materials or FGPMs is a rather involved problem. The situation is even more formidable when non-homogeneity has to be considered. It is therefore not surprising that only scant information regarding transient wave propagation problems has been presented. As in

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http://dx.doi.org/10.1016/j.ijsolstr.2014.09.021
0020-7683/

the case of the Stoneley wave, whose mechanical displacements are in the sagittal plane, the amplitude of this wave decreases with the distance away from the interface into both media (Stoneley, 1924). Bleustein (1968) and Gulayaev (1969) simultaneously discovered that there exists a shear horizontal (SH) electro-acoustic surface mode in a class of transversely isotropic piezoelectric media, which is known today as the BG wave. The BG wave is a unique result in the repertoire of surface acoustic wave (SAW) theory because it has no counterpart in purely elastic solids. As a matter of fact, since its discovery, the BG wave theory has become one of the cornerstones of modern electro-acoustic technology. It was shown that a BG wave can exist in cubic crystals of $\bar{4}3m$ and 23 classes, along [1 1 0] direction on the $(\bar{1}\,1\,0)$ plane and their equivalent orientations. The velocity equations for piezoelectric and elastic surface waves were derived and their characteristics were discussed by Tseng (1970). A pure shear elastic surface wave (MT wave) can propagate along the interface of two identical crystals, in class 6 mm, when the z-axes of these crystals, both parallel to the interface and perpendicular to the propagation direction, are in opposite directions (Maerfeld and Tournois, 1971). The general equations and the fundamental piezoelectric matrix derived for the anti-plane wave motion and Floquet theory were applied to obtain the passing and stopping bands in a periodically layered infinite space by Honein and Herrman (1992). Taking into account both the optical effect as well as the contribution from the rotational part of an electric field, the obtained solutions not only were valid for any wave speed range but also provided accurate formulas to evaluate the acousto-optic interaction due to piezoelectricity. As the wave speed is much less than the speed of light, the solution degenerates to the well-known BG wave or MT wave (Li, 1996). The surface acoustic wave (SAW) can be excited and detected efficiently by using an interdigital transducer (IDT) placed on a piezoelectric substrate. Therefore, a vast amount of effort was invested in the research and development of SAW devices for military and communication applications, such as delay lines and filters for radar. The propagation mode in most devices is the Rayleigh wave on a free surface of a piezoelectric substrate. Recently, Ma et al. (2007) used Cagniard's method to construct the full-field transient solutions of piezoelectric bi-materials. Each term represents a physical transient wave. The existence condition of the surface wave of piezoelectric bi-materials is restricted to the situation where the shear wave speeds of the two piezoelectric materials are very close.

Laplace transform and the inversion of Laplace transform are usually used for analyzing the dynamic wave-propagation problem. However, using the analytical inversion of Laplace transform for analysis renders the mathematical composition excessively complex and difficult; therefore, it is only suitable for relatively simple geometric structures. The numerical inversion of Laplace transform is more practical for calculating complex problems. Lin and Ma (2011) used the numerical Laplace inversion and the Durbin method to obtain the transient stress responses for a 20-layered elastic structure. Ma et al. (2012) used the Durbin method to analyze the long-time responses for a functionally graded slab. Lin and Ma (2012) found that the transient responses of the continuously distributed multilayered media can be simulated by the effective material concept and are applicable to analyze the wave propagation problem in a functionally graded slab. Ing et al. (2013) presented an extended Durbin method for a two-sided Laplace inversion, and evaluated the transient stress and electric displacement of a two-layered piezoelectric medium.

In this study, the transient wave propagation problem of an FGPM slab was investigated. The slab was subjected to dynamic point loading on the top surface. The transient response was analyzed by employing the Laplace transform method. The interface and boundary conditions were used to construct the system of equations for determining the field vectors in the FGPM slab. Subsequently, the Durbin method was used to implement the double inversions for the transient response of wave propagation in the FGPM slab.

2. Statement of the problem

For a linear FGPM medium, the constitutive equations can be expressed as

$$\sigma_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k, \tag{1}$$
$$D_i = e_{ikl}S_{kl} + \varepsilon_{ik}E_k, \tag{2}$$

where σ_{ij}, S_{kl}, E_k and D_i indicate the stress tensor, strain tensor, electric field vector, and electric displacement vector, respectively. In the absence of body forces, the governing equations can be expressed as

$$\sigma_{ij,j} = \rho\ddot{u}_i, \tag{3}$$
$$D_{i,i} = 0, \tag{4}$$

where ρ is the mass density, u_i is the displacement vector, and the superposed dot indicates material differentiation with respect to time. The material property parameters of the FGPM are assumed to vary exponentially along the y-direction according to the following exponential law

$$c_{44} = c_{440}e^{\beta y}, \quad e_{15} = e_{150}e^{\beta y}, \quad \varepsilon_{11} = \varepsilon_{110}e^{\beta y}, \quad \rho = \rho_0 e^{\beta y}, \tag{5}$$

where c_{44}, ε_{11}, and e_{15} denote the elastic modulus, dielectric permittivity, and piezoelectric constant, respectively. β represents the gradient coefficient of the FGPM. To reduce the complexity of mathematical analysis, all of the physical properties were assumed to vary in the same way. Consider the following anti-plane displacement and electric potential fields:

$$u_1 = u_2 = 0, \quad u_3 = w(x, y, t), \quad \phi = \phi(x, y, t). \tag{6}$$

From the constitutive relations of FGPM poled in the z-direction, the nontrivial components of stresses and electric displacements are

$$\tau_{xz} = c_{440}e^{\beta y}\frac{\partial w}{\partial x} + e_{150}e^{\beta y}\frac{\partial \phi}{\partial x}, \tag{7}$$
$$\tau_{yz} = c_{440}e^{\beta y}\frac{\partial w}{\partial y} + e_{150}e^{\beta y}\frac{\partial \phi}{\partial y}, \tag{8}$$
$$D_x = e_{150}e^{\beta y}\frac{\partial w}{\partial x} - \varepsilon_{110}e^{\beta y}\frac{\partial \phi}{\partial x}, \tag{9}$$
$$D_y = e_{150}e^{\beta y}\frac{\partial w}{\partial y} - \varepsilon_{110}e^{\beta y}\frac{\partial \phi}{\partial y}. \tag{10}$$

Substituting Eqs. (5)–(10) into Eqs. (3) and (4), the governing equations can be obtained as follows:

$$c_{440}\nabla^2 w + e_{150}\nabla^2\phi + \beta c_{440}\frac{\partial w}{\partial y} + \beta e_{150}\frac{\partial \phi}{\partial y} = \rho_0\ddot{w}, \tag{11}$$
$$e_{150}\nabla^2 w - \varepsilon_{110}\nabla^2\phi + \beta e_{150}\frac{\partial w}{\partial y} - \beta\varepsilon_{110}\frac{\partial \phi}{\partial y} = 0, \tag{12}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{13}$$

is the two-dimensional Laplacian. It is worth noting that Eqs. (13) and (12) are two coupled partial differential equations. By introducing a function

$$\psi = \phi - \frac{e_{150}}{\varepsilon_{110}}w, \tag{14}$$

then the constitutive equations are reduced to the following forms: