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From infinitesimal to full contact between rough surfaces: Evolution of the contact area

Vladislav A. Yastrebov [a,*], Guillaume Anciaux [b], Jean-François Molinari [b]

[a] MINES ParisTech, PSL Research University, Centre des Matériaux, CNRS UMR 7633, BP 87, F 91003 Evry, France

[b] Computational Solid Mechanics Laboratory (LSMS, IIC-ENAC, IMX-STI), Ecole Polytechnique Fédérale de Lausanne (EPFL), Bât. GC, Station 18, CH 1015 Lausanne, Switzerland

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ABSTRACT

We carry out a statistically meaningful study on self-affine rough surfaces in elastic frictionless non-adhesive contact. We study the evolution of the true contact area under increasing squeezing pressure from zero up to full contact, which enables us to compare the numerical results both with asperity based models at light pressures and with Persson's contact model for the entire range of pressures. A good agreement of numerical results with Persson's model is obtained for the shape of the area-pressure curve especially near full contact, however, we obtain qualitatively different results for its derivative at light pressures. We investigate the effects of the longest and shortest wavelengths in surface spectrum, which control the surface Gaussianity and spectrum breadth (Nayak's parameter). We revisit the influence of Nayak's parameter, which is frequently assumed to play an important role in mechanics of rough contact.

1. Introduction

Contact, adhesion and friction play an important role in many natural (e.g., earthquakes) and engineering systems, for example, assembled parts in engines, railroad contacts, bearings and gears, breaking systems, tire-road contacts, metal forming, vehicle crash, bio mechanics, granular materials, electric contacts, liquid sealing, etc. In all these examples, the contacting surfaces are rough. Being in dry contact (in absence of lubrication) means that the contacting solids touch each other at many separate spots, whose area may be drastically different from the prediction of classical Hertz's contact theory. This roughness and complexity of the contact interface may be a major factor in analysis of such systems for strength, critical stresses, fatigue and damage, fracture initiation, friction, adhesion, wear, heat and electric charge transfer, and percolation. Real or true contact area is one of the central characteristics of the contact between rough surfaces. In this paper we analyze by means of numerical analysis how the real contact area changes with applied pressure and what are the relevant properties of the surface roughness that influence this evolution. The numerical results are compared with existing analytical models and numerical results of other authors. We consider the problem of rough contact in its simplest formulation: frictionless and non-adhesive contact between linearly elastic half-spaces. Regardless of the apparent simplicity of the problem and multiple analytic/experimental/numerical studies, many questions remain open.

1.1. Roughness

All surfaces in nature and industry are rough under certain magnification. This roughness possesses specific characteristics. Most of rough surfaces are self-similar or self-affine, i.e. the roughness scales under magnification with a given scaling coefficient all along the magnification range from macroscopic down to nanometric scales. Typical examples of this scaling are found in Earth landscapes, coast line, tectonic faults, ocean's surface and engineering surfaces (Thomas, 1999; Meakin, 1998). Among a wide variety of rough surfaces, the class of isotropic Gaussian surface deserves a particular attention from the scientific community due to its relative simplicity and generality (Longuet-Higgins, 1957; Nayak, 1971; Greenwood and Williamson, 1966; Bush et al., 1975). By isotropy one implies that statistical properties of any two profiles measured along different directions are identical. By normality or Gaussianity of a surface one implies that surface heights are normally distributed.

The self-affinity of rough surfaces may be decoded by analysis of its autocorrelation function $R(x,y)$ or the Fourier transform of R which is called the power spectral density (PSD) $\Phi(k_x,k_y)$, where

* Corresponding author. Tel.: +33 1 60 76 31 53.
E-mail addresses: vladislav.yastrebov@mines-paristech.fr (V.A. Yastrebov), guillaume.anciaux@epfl.ch (G. Anciaux), jean-francois.molinari@epfl.ch (J.-F. Molinari).
URL: http://www.yastrebov.fr (V.A. Yastrebov).

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$k_x,\ k_y$ are the wavenumbers[1] in orthogonal directions $x,\ y$. For many natural and engineering surfaces, the PSD decays as a power-law of the wavenumber (Majumdar and Tien, 1990; Dodds and Robson, 1973; Vallet et al., 2009):

$$\Phi(k_x, k_y) \sim \left[\sqrt{k_x^2 + k_y^2}\right]^{-2(1+H)},$$

where H is the Hurst roughness exponent which is related to the fractal dimension D as $D = 3 - H$. The PSD is bounded at the upper scale by the longest wavelength λ_l (or the smallest wavenumber $k_l = L/\lambda_l$). To handle a continuum model of a rough surface, the PSD may be bounded at the lower scale by the shortest wavelength λ_s (or the highest wavenumber $k_s = L/\lambda_s$) (for detailed discussion see Section 3).

1.2. Mechanics of rough contact

The surface roughness has important consequences on the mechanics and physics of contact. For instance, the widely used Hertz theory of contact (Hertz, 1882; Johnson, 1987), in which the contacting surfaces are assumed to be smooth, is not valid for rough surfaces, as the roughness induces high fluctuations of local deformations close to the contact surface, that go easily beyond the elastic limits and/or fracture strength of materials. This fact follows directly from the observation, that for most materials and loads the real contact area A between contacting solids is only a small fraction of the nominal (apparent) contact area A_0 predicted by Hertz theory. The real contact area characterizes the transfer of heat and electricity through the contact interface, frictional properties of the contact as well as the strength of adhesion and amount of wear.

The stochastic nature of rough contact makes it difficult to estimate material rupture or stick–slip transition within a deterministic approach and requires a probabilistic description and a statistical analysis. Many factors affect the mechanics of rough contact. For example, the real contact area depends on mechanics (contact pressure, friction, adhesion, wear), on multi-physics effects (Joule heating in electric contact, chemical reactions, frictional heating), on time (viscosity and aging of materials) and environment (oxidation of surfaces, temperature, humidity). While in experiments it is hard to study all these aspects separately and deduce the more relevant ones, in numerical simulations it is difficult to include many mechanisms to study their combined effect, as the models become excessively complex and hardly verifiable. In experiments, the contacting surfaces are also hard to observe *in situ* to characterize directly the contact zones. Thus indirect observation methods were adapted (measurements of the heat and electric transfer through the contact interface, Bowden and Tabor, 2001), which may bias the measurements due to the strong coupling between involved phenomena.

Another challenge in rough contact arises from the breakdown of continuum contact mechanics at nano-scale (Luan and Robbins, 2005). This issue is relevant if the roughness is present at atomic scale (Krim and Palasantzas, 1995), which is often the case (Misbah et al., 2010; Einax et al., 2013) particularly for crystalline materials for which dislocations reaching free surfaces leave atomic "steps" on them. This atomic roughness can be taken into account by means of atomic modeling (Sinnott et al., 2008; Spijker et al., 2013). But it is particularly hard to link the atomistic simulations of rough surfaces with macroscopic results as there is a lack of representativity in analyzed samples. As there is no scale separation in surface roughness, classical hierarchical homogenization models cannot be directly applied to the analysis of rough contact. However, coupling between atomistic simulations with finite element models (Ramisetti et al., 2013; Ramisetti et al., 2014) (eventually accompanied with discrete dislocation dynamics coupling) is a promising technique to perform large simulations of contact between rough surfaces at atomic scale (Anciaux and Molinari, 2009; Anciaux and Molinari, 2010).

To remain in the framework of continuum mechanics, one needs to abandon the atomistic scale and introduce an artificial short wavelength cutoff λ_s in the surface to obtain a roughness which is smooth under a certain magnification. Consideration of such surfaces with truncated fractality lies in the foundation of classical analytical models of rough contact; moreover, valid numerical studies are only possible on surfaces which are smooth enough. Normally, at the longest wavelengths, real surfaces do not demonstrate self-affinity and the PSD has a plateau for a certain range of wavelengths (Persson et al., 2005). This plateau includes wavelengths from λ_l to λ_r, where λ_l is the longest wavelength and λ_r is a so-called rolloff wavelength. So the fractality of rough surfaces is truncated at certain high and low frequencies.

We consider normal contact between two linearly elastic half-spaces ($E_1,\ \nu_1$ and $E_2,\ \nu_2$ are Young's moduli and Poisson's ratios of the solids) possessing rough surfaces $h_1(x,y),\ h_2(x,y)$. Under assumption of frictionless non-adhesive contact, this problem may be replaced by contact between a rigid surface with an effective roughness $h = h_1 - h_2$ and an elastic flat half-space with effective Young's modulus (Johnson, 1987)

$$E^* = E_1E_2/((1-\nu_1^2)E_2 + (1-\nu_2^2)E_1). \quad (1)$$

This substitution is common and enables to use numerical methods, which are simpler than those needed for the original formulation.

In Section 2 we give an overview of analytical and numerical models of rough contact. In Section 3 we discuss the generation of rough surfaces with prescribed properties, also we demonstrate the role of cutoffs in surface spectrum on the Gaussianity of resulting roughness. Equations linking Nayak's parameter and asperity density with the Hurst exponents and cutoff wavenumbers are derived (see also A). In Section 4 the numerical model and the set-up are briefly outlined. The evolution of the real contact area at light loads is analyzed and compared to analytical models in Section 5. General trends in the contact area evolution from zero to full contact are discussed in Section 6. Asymptotics of the contact area near the full contact is investigated in Section 7. In Section 8 we propose an estimation of error bounds of the contact area in numerical simulations and experimental measurements. In Section 9 we discuss the obtained results and prospective work.

2. Overview of mechanical models of rough contact

2.1. Analytical models

Two classes of analytical models exist. The first class is based on the notion of asperities (summits of the surface at which $\nabla h = 0$). The pioneering work by Greenwood and Williamson (GW) (1966) was followed by more elaborated models refining geometrical and statistical aspects of the GW models (McCool, 1986; Bush et al., 1975; Greenwood, 2006; Thomas, 1999; Carbone and Bottiglione, 2008). The statistical properties of asperities (e.g., joint probability density of heights and curvatures) are often derived from the random process description of rough surfaces (Nayak, 1971) or may be measured directly; tips of asperities may be assumed spherical or elliptical, with constant or varying curvature. Note that the progress in asperity based models is strongly associated with Nayak's extension (Nayak, 1971) of Longuet-Higgins studies (Longuet-Higgins, 1957) on statistical properties of random

[1] Hereinafter by a wavenumber we imply a spectroscopic wavenumber normalized by the sample length L to render them dimensionless.